$\boxed{V1}$

1) a) Autov. 1, 2 $\quad \mu(1)=2 \quad \mu(2)=1$

b) $\lambda=1 \quad A-I=\begin{pmatrix} 1 & 0 & -3 \\ 1 & 0 & -3 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow U_1 = L\left(\begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}3\\0\\1\end{pmatrix}\right)$

$\lambda=2 \quad A-2I=\begin{pmatrix} 0 & 0 & -3 \\ 1 & -1 & -3 \\ 0 & 0 & -1 \end{pmatrix} \Rightarrow U_2 = L\left(\begin{pmatrix}1\\1\\0\end{pmatrix}\right)$

---

2) a) $T(1,0,0) = \frac{1}{2}T\left((1,0,1) + T(1,0,-1)\right) = \left(\frac{3}{2}; \frac{1}{2}; -\frac{1}{2}\right)$

(pp)

$T(0,1,0) = -\frac{1}{2}T(0,-2,0) = (-1,0,-2)$

$T(0\,0\,1) = \frac{1}{2}\left(T(1\,0\,1) - T(1\,0\,-1)\right) = \left(\frac{1}{2}; \frac{1}{2}; \frac{1}{2}\right)$

$\Rightarrow A = \begin{pmatrix} 3/2 & -1 & \frac{1}{2} \\ 1 & 0 & 0 \\ -1 & -2 & 1 \end{pmatrix}$

b) $\ker T: \begin{pmatrix} 3 & -2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 1 \end{pmatrix} \rightarrow \ker T = L\left(\begin{pmatrix}0\\1\\2\end{pmatrix}\right)$

$\operatorname{Im} T = L\left(\begin{pmatrix}1\\0\\2\end{pmatrix}; \begin{pmatrix}3\\2\\-2\end{pmatrix}\right)$

---

2) a) $q(x,y,z) = 2x^2 + (\lambda+1)y^2 - 2yz + 2z^2$

(parz)

Conica ass ad A è un'iperbole $\Leftrightarrow A_{00} < 0 \quad (e \det A \neq 0)$

$\Leftrightarrow 2\cdot(\lambda+1) - 1 < 0 \Leftrightarrow 2\lambda+1 < 0 \Leftrightarrow \boxed{\lambda < -\frac{1}{2}}$

b) $\lambda < -\frac{1}{2} \rightarrow (2,1)$

$\lambda > -\frac{1}{2} \rightarrow (3,0)$

$\lambda = -\frac{1}{2} \rightarrow A_{00} = \begin{pmatrix} \frac{1}{2} & -1 \\ -1 & 2 \end{pmatrix} \xrightarrow{O_A} (2,0)$

---

3) a) $r: \rho\begin{vmatrix} x-2 & 1 \\ y-1 & -2 \\ z & 1 \end{vmatrix} = 1 \rightarrow \begin{cases} -2x+4-y+1=0 \\ x-2-z=0 \end{cases} \rightarrow \begin{cases} 2x+y-5=0 \\ x-z-2=0 \end{cases}$

b) 2 vett di $\vec{\pi}$ ortog. $\quad v_1 = (1,0,-1) \quad$ perché $\vec{\pi}: v_x - 2v_y + v_z = 0$

$v_2 = (1,1,1)$

$\longrightarrow \quad r_1: \begin{cases} x = 2+\alpha \\ y = 1 \\ z = -\alpha \end{cases} \alpha \in \mathbb{R}; \quad r_2: \begin{cases} x = 2+\alpha \\ y = 1+\alpha \\ z = \alpha \end{cases} \alpha \in \mathbb{R}$

$\boxed{V2}$

1) a) Autoval: $2, 4$ $\quad \mu(2) = 1 \quad \mu(4) = 2$

b) $\lambda = 2 \to A - 2I = \begin{pmatrix} 0 & 0 & 3 \\ -2 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix} \to U_2 = L\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$\lambda = 4 \to A - 4I = \begin{pmatrix} -2 & 0 & 3 \\ -2 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix} \to U_4 = L\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}\right)$

---

2) (APPELLO) a) $T(1,0,0) = \frac{1}{2}\left(T(1,1,0) + T(1,-1,0)\right) = (2, 0, 2)$

$T(0,1,0) = \frac{1}{2}\left(T(1,1,0) - T(1,-1,0)\right) = (0, -1, -1)$

$T(0,0,1) = -\frac{1}{2}\left(T(0,0,-2)\right) = (0, -1, -1)$ $\Rightarrow A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & -1 \\ 2 & -1 & -1 \end{pmatrix}$

b) $\ker T : \begin{cases} x = 0 \\ y + z = 0 \end{cases} \to \ker T = L\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

$\operatorname{Im} T = L\left(\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}; \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right)$

---

2) (2° parz) a) $A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & \lambda-1 & 2 \\ 0 & 2 & 3 \end{pmatrix}$

$\mathcal{C}_A : 3 + (\lambda-1)x^2 + 4xy + 3y^2 = 0$

$\mathcal{C}_A$ è un'iperbole

$\Leftrightarrow A_{00} = 0 \Leftrightarrow 3(\lambda-1) - 4 < 0 \Leftrightarrow \lambda < \frac{7}{3}$

$3\lambda - 7 < 0$

b) $\lambda < \frac{7}{3} \to \sigma_A = (2, 1)$

$\lambda > \frac{7}{3} \to \sigma_A = (3, 0)$

$\lambda = \frac{7}{3} \to \sigma_A = (2, 0)$

---

3) a) $r : \rho\begin{pmatrix} x-2 & 2 \\ y+1 & -2 \\ z-1 & 0 \end{pmatrix} = 1 \to \begin{cases} \begin{vmatrix} x-2 & -1 \\ y+1 & 1 \end{vmatrix} = 0 \\ \begin{vmatrix} x-2 & 1 \\ z-1 & 0 \end{vmatrix} = 0 \end{cases} \quad \begin{cases} x + y + 1 = 0 \\ z - 1 = 0 \end{cases}$

b) 2 vettori di $\vec{\pi} : (v_x - v_y = 0)$ ortogonali tra loro:

$v_1 = (1, 1, 0) \quad v_2 = (0, 0, 1)$

$\to r_1 : \begin{cases} x = 2 + \alpha \\ y = -1 + \alpha \\ z = 1 \end{cases} \quad (\alpha \in \mathbb{R}) \qquad r_2 : \begin{cases} x = 2 \\ y = -1 \\ z = 1 + \alpha \end{cases} \quad (\alpha \in \mathbb{R})$

V3

1) a) Autoval: $1, -1$ $\quad \mu_a(1)=1 \quad \mu(-1)=2$

b) $\lambda=1 \to A-I = \begin{pmatrix} 0 & 0 & 5 \\ 2 & -2 & 5 \\ 0 & 0 & -2 \end{pmatrix} \to U_1 = L\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$\lambda=-1 \to A+I = \begin{pmatrix} 2 & 0 & 5 \\ 2 & 0 & 5 \\ 0 & 0 & 0 \end{pmatrix} \to U_2 = L\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}\right)$

---

2) I app.)

a) $T(100) = \frac{1}{2}\left(T(101) - T(-101)\right) = (-1, -1, \frac{1}{2})$

$T(0,1,0) = \frac{1}{3}(333) = (1,1,1)$

$T(001) = \frac{1}{2}\left(T(-1,0,1) + T(101)\right) = (0,0,\frac{1}{2})$

$\to A = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 1 & 0 \\ 1/2 & 1 & 1/2 \end{pmatrix}$

b) $\operatorname{Ker} T: \begin{cases} x-y=0 \\ \frac{1}{2}x+y+\frac{1}{2}z=0 \end{cases} \to \operatorname{Ker} T = L\begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$

$\operatorname{Im} T = L\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right)$

---

2) parz)

a) $q(x,y,z) = x^2 + 2y^2 + 4yz + (1-\lambda)z^2$

$\mathcal{C}_A$ iperbole $\iff A_{00} < 0 \iff 2(1-\lambda)-4<0 \to \boxed{\lambda > -1}$

$1-\lambda-2<0$

b) $\lambda > -1 \to \sigma_A = (2,1)$

$\lambda < -1 \to \sigma_A = (3,0)$

$\lambda = -1 \to \sigma_A = (2,0)$

---

3) a) r : $\rho\begin{pmatrix} x & 1 \\ y-1 & -1 \\ z-3 & 2 \end{pmatrix} = 1 \to \begin{cases} -x-y+1=0 \\ 2x-z+3=0 \end{cases}$

b) 2 vettori $v_1\, v_2 \in \vec{\pi}$: $v_x - v_y + 2v_z = 0$, $\quad v_1 \perp v_2$

$v_1 = (1,1,0) \qquad v_2 = (1,-1,-1)$

$\to r_1 \begin{cases} x = \alpha \\ y = 1+\alpha \\ z = 3 \end{cases} (\alpha \in \mathbb{R}) \qquad r_2 : \begin{cases} x = \alpha \\ y = 1-\alpha \\ z = 3-\alpha \end{cases} (\alpha \in \mathbb{R})$

$\boxed{V4}$

1) a) Autoval: -1 con m.a 1 $\quad \Delta_A(t) = (t+1)(t+2)^2$
-2 con m.a 2

b) $\lambda = -1 \rightarrow A + I = \begin{pmatrix} 0 & 0 & 3 \\ 1 & -1 & -3 \\ 0 & 0 & -1 \end{pmatrix} \rightarrow U_{-1} = L\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$\lambda = -2 \rightarrow A + 2 = \begin{pmatrix} 1 & 0 & 3 \\ 1 & 0 & -3 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow U_{-2} = L\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

---

2) (app.) a) $T(1,0,0) = \frac{1}{6} T(6,0,0) = (2, 0, -1)$

$T(0,1,0) = \frac{1}{2}\left(T(0,1,1) - T(0,-1,1)\right) = (0, 0, -\frac{3}{2}) \rightarrow A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & -\frac{3}{2} & -\frac{1}{2} \end{pmatrix}$

$T(0,0,1) = \frac{1}{2}\left(T(0,-1,1) + T(0,1,1)\right) = (1, 0, -\frac{1}{2})$

b) $\operatorname{Ker} T: \begin{cases} 2x + z = 0 \\ x + \frac{3}{2}y + \frac{1}{2}z = 0 \end{cases} \rightarrow \begin{cases} 2x + z = 0 \\ y = 0 \end{cases} \rightarrow \operatorname{Ker} T = L\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$

$\operatorname{Im} T = L\left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right)$

---

2) (2' parz) a) $A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & -2 & 2-\lambda \end{pmatrix}$ La conica C_A è un'iperbole $\Leftrightarrow A_{00} < 0 \Leftrightarrow 2 - \lambda - 4 < 0$ $\Leftrightarrow \boxed{\lambda > -2}$

b) $\lambda > -2 \rightarrow \sigma_A = (1, 2)$
$\lambda < -2 \rightarrow \sigma_A = (2, 1)$
$\lambda = -2 \rightarrow \sigma_A = (1, 1)$

---

3) a) $r: \rho\begin{pmatrix} x+2 & 2 \\ y & -1 \\ z-1 & 1 \end{pmatrix} = 1 \rightarrow r: \begin{cases} x + 2 + 2y = 0 \\ x + 2 - 2z + 2 = 0 \end{cases} \quad \begin{cases} x + 2y + 2 = 0 \\ x - 2z + 4 = 0 \end{cases}$

b) 2 vett. v_1, v_2, $v_1 \perp v_2$, $v_1, v_2 \in \vec{\pi}: 2v_x - v_y + v_z = 0$
$v_1 = (1, 2, 0) \qquad v_2 = (2, -1, -5)$

$r_1: \begin{cases} x = -2 + \alpha \\ y = 2\alpha \\ z = 1 \end{cases} \ (\alpha \in \mathbb{R}) \qquad r_2: \begin{cases} x = -2 + 2\alpha \\ y = -\alpha \\ z = 1 - 5\alpha \end{cases} \ (\alpha \in \mathbb{R})$