Ornstein-Zernike behaviour for the correlation functions of the ground state of the quantum Ising model with transverse field. M.C., M. Gianfelice

$$H = -\sum J_{|x-y|} \sigma_x^{(3)} \sigma_y^{(3)} - h \sum_x \sigma_x^{(1)}$$

$$J_r = 0 \quad \text{for } r > K,$$

$$\Lambda \subset \mathbb{Z}^d \qquad \mathcal{H} = \bigotimes_{x \in \Lambda} \mathbb{C}^2$$

$\langle \ \rangle_\Lambda$ ground state.

Main result

$$\langle \ \rangle = \lim_{\Lambda \uparrow \infty} \langle \ \rangle_{\Lambda}$$

Let $h_c = \inf \{ h \mid \lim_{x \to \infty} \langle \sigma_0 \sigma_x \rangle = 0 \}$.

Then for $h > h_c$ the following limit exists $\forall$ all $x \in \mathbb{R}^d$

$$\lim_{n \to \infty} -\frac{1}{n} \log \langle \sigma_0 \sigma_{[nx]} \rangle \triangleq \tau(x)$$

$\tau(x)$ is a norm. Its unit sphere is strictly convex, with strictly positive Gaussian curvature, real analytic.

$$\langle \sigma_0 \sigma_x \rangle = \varphi\left(\frac{x}{|x|}\right) \frac{1}{|x|^{\frac{d}{2}}} (1 + o(1))\, e^{-\tau(x)}.$$

("Ornstein-Zernike" behaviour.

the random line representation of two-point correlation function.

$B \subset\subset \mathcal{E}$ $\quad V_B \triangleq \{x \in \mathbb{Z}^d \mid \exists\, e \in B \text{ with } x \in e\}$

$x \in V_B$ $\quad$ index of x in B

$$\mathrm{ind}(x, B) \triangleq \sum_{e \in B} I_{\{x \in e\}}$$

boundary of B

$$\partial B \triangleq \{x \in V_B \mid \mathrm{ind}(x, B) \text{ is odd}\}$$

we fix an ordering for the edges incident in x and we write $e \leq e'$.

We can write

$$e^{\beta J(e)\sigma_x \sigma_y} = \cosh(\beta J(e))\left(1 + \sigma_x \sigma_y \tanh(\beta J(e))\right)$$ and

obtain that

$$\langle \sigma_x \sigma_y \rangle_{B,\beta} = Z_\beta(B)^{-1} \sum_{\substack{D \subset B \\ \partial D = \{x, y\}}} \prod_{e \in D} \tanh(\beta J(e))$$

Representation

I interval $I \subset \mathbb{R}$. X_I space of functions from I to $\{-1, 1\}$. μ_I is the probability measure on X_I obtained from a Poisson point process with intensity h, where the points of the process represent where the function switches value and μ_I is assumed to be invariant under sign inversion. Given Λ ~~interval~~ $\subset \mathbb{Z}^d$ ~~$\subset \mathbb{Z}^d$~~ $\Lambda \subset \mathbb{Z}^d$, we define the ~~$\Delta$ (interval~~ Gibbs measure on $X^{\Lambda}_{[-\frac{\beta}{2}, \frac{\beta}{2}]}$ with density

$$Z^{-1} \exp\left(-J \sum_{\langle x,y \rangle} \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} \sigma_x(t)\sigma_y(t)\,dt\right)$$

with respect to $\bigotimes_{x \in \Lambda} \mu_{[-\frac{\beta}{2}, \frac{\beta}{2}]}$.

This measure allows to represent $e_m(\beta)$ and e_m with $\Lambda \equiv \Delta_m$.

Quantum Ising model on $\mathbb{Z}^d$ as limit of classical Ising models on $\mathbb{Z}^{d+1}$.

The ground state of the quantum Ising model with transverse field can be obtained as limit of classical Ising models on meshes. We consider in this limit the random line representation of two-point correlation function.

We get as a limit a measure on trajectories from x to y where the ~~va~~ vertical lines have exponential distribution and each horizontal segment gives a factor J.

The random line representation of two-point correlation functions for the Ising model.

B set of bonds $\quad V_B \triangleq \{x \in \mathbb{Z}^d \mid \exists e \in B \text{ with } x \in e\}$

$x \in V_B$ index of x in B

$$\operatorname{ind}(x, B) \triangleq \sum_{e \in B} I_{\{x \in e\}}$$

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$$\partial B \triangleq \{x \in V_B \mid \operatorname{ind}(x, B) \text{ is odd}\}$$

Ordering for the edges incident in x

$e \leq e'$

$$e^{\beta J(e) \sigma_x \sigma_y} = \cosh(\beta J(e))(1 + \sigma_x \sigma_y \tanh(\beta J(e)))$$

$$\langle \sigma_x \sigma_y \rangle_{B,\beta} = Z_\beta(B)^{-1} \sum_{\substack{D \subset B \\ \partial D = \{x, y\}}} \prod_{e \in D} \tanh(\beta J(e))$$

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From $D \subset B$ with $\partial D = \{x, y\}$, we want to extract a self-avoiding path.

Step 1. Set $z'_0 = y$, $j = 0$, $\Delta_0 = \emptyset$.

Step 2. Let $e'_j = (z'_j, z'_{j+1})$ be the first edge in $B'_{z'_j} \setminus \Delta_j$ such that $e'_j \in D$.

This defines z'_{j+1}.

Step 3. Set $\Delta_{j+1} = \Delta_j \cup \{e \in B_{z'_j} \mid e \leq e'_j\}$.

If $z'_{j+1} = x$, then set $n = j+1$ and stop.

Otherwise update $j \mapsto j+1$ and return to Step 2.

This procedure produces a sequence $(z'_0, \ldots, z'_n)$. Let $z_k \triangleq z'_{n-k}$ and $e_k \triangleq e'_{n-k}$. We have constructed a path $\lambda \triangleq \lambda(D) \triangleq (z_0 = x, \ldots, z_n = y)$ such that $(z_i, z_{i+1}) \in D$ $i = 0, \ldots, n-1$

$(z_i, z_{i+1}) \neq (z_j, z_{j+1})$ for $i \neq j$

$$\Delta_\lambda \triangleq \Delta_n = \bigcup_{i=1}^{n} \{e \in B_{z_i} \mid e \leq e_i\}$$

For any $D \subset B$ with $\partial D = \{x, y\}$, $\lambda(D) = \lambda$ if and only if (considering λ as a set of edges) λ and $(\Delta(\lambda) \setminus \lambda) \cap D = \phi$.

We can therefore write

$$\langle \sigma_x \sigma_y \rangle_{B,\beta} = \sum_{x \to y} q_{B,\beta}(\lambda) \text{ where,}$$

writing $\quad w(\lambda) = \prod_{e \in \lambda} \tanh(\beta J(e))$

and $$q_{B,\beta}(\lambda) = w(\lambda) \frac{Z_\beta(B \setminus \Delta(\lambda))}{Z_\beta(B)}$$

$$H = -h(\sigma^{(1)} - 1)$$

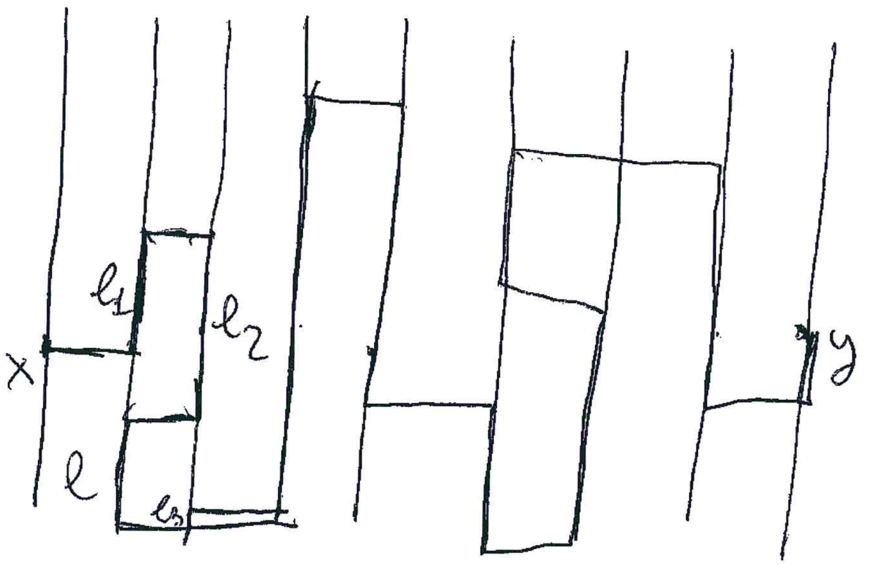
$$\langle \sigma^{(3)} = \omega'' | e^{-tH} | \sigma^{(3)} = \omega' \rangle =$$

$$= \frac{1}{2}(1 - e^{-4th})^{\frac{1}{2}} e^{K\omega''\omega'}$$

$$e^{-2K} = \tanh(th)$$

$$K = -\frac{1}{2}\log(\tanh(th))$$

$$\tanh(K) = \frac{1 - \tanh(th)}{1 + \tanh(th)} = 1 - 2th + o(t)$$



$$e^{-2h(\sum l_i)} J^N$$

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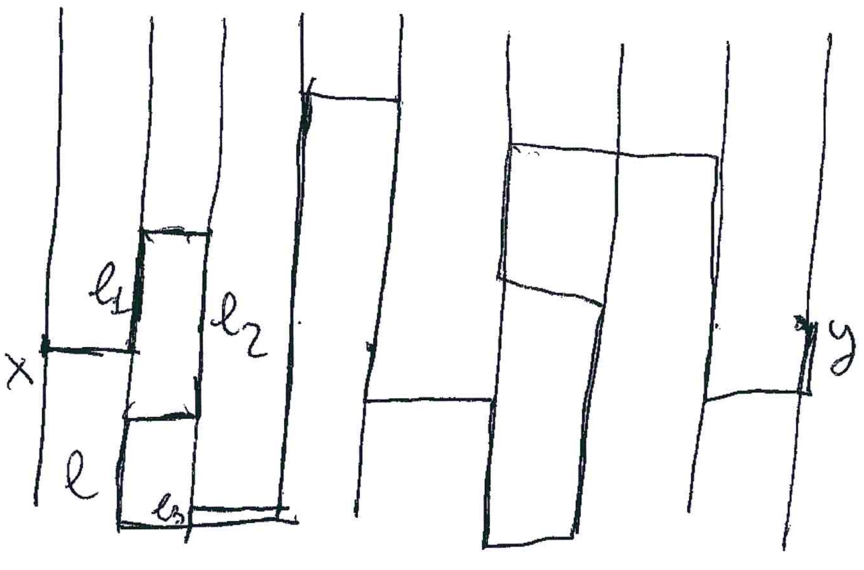

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Random walk representation