ANALISI - CESENA (MODULO II) ①

(1) Ⓐ $f(x,y) = \ln(x^2+3y^2)\cos(x+y)$ $P=(\frac{\pi}{2}, 0)$

$f(\frac{\pi}{2},0) = \log(\frac{\pi^2}{4}) \cdot 0 = \emptyset$

$\triangleright\ \partial_x f(x,y) = \frac{2x}{x^2+3y^2}\cos(x+y) - \ln(x^2+3y^2)\cdot \operatorname{sen}(x+y)$

$\to \partial_x f(\pi/2, 0) = \emptyset - \ln(\frac{\pi^2}{4}) = -2\log(\pi/2)$

$\triangleright\ \partial_y f(x,y) = \frac{6y}{x^2+3y^2}\cdot\cos(x+y) - \ln(x^2+3y^2)\operatorname{sen}(x+y)$

$\to \partial_y f(\pi/2, 0) = -2\ln(\pi/2)$

EQ. PIANO TANGENTE

$$z = -2\ln \pi/2\ (x - \pi/2) - 2\ln \pi/2\ y$$

$$= -2\ln \pi/2\ (x + y - \pi/2)$$

B) $f(x,y) = e^{2x+y} \arctan y \qquad P = (1, \pi/4)$

$$f(1, \pi/4) = e^{2+\pi/4} \cdot \arctan \pi/4 = e^{2+\pi/4}$$

$$\partial_x f(x,y) = 2e^{2x+y} \arctan y$$

$$\rightarrow \partial_x f(1, \pi/4) = 2e^{2+\pi/4}$$

$$\cdot\ \partial_y f(x,y) = e^{2x+y}\left(\arctan y + \frac{1}{1+y^2}\right)$$

$$\rightarrow \partial_y f(1, \pi/4) = e^{2+\pi/4}\left(1 + \frac{1}{1+\pi^2/16}\right)$$

$$= e^{2+\pi/4}\left(1 + \frac{16}{16+\pi^2}\right) = e^{2+\pi/4}\left(\frac{32+\pi^2}{16+\pi^2}\right)$$

PIANO TANGENTE

$$z = e^{2+\pi/4} + 2e^{2+\pi/4}(x-1) + e^{2+\pi/4}\left(\frac{32+\pi^2}{16+\pi^2}\right)\cdot(y-\pi/4)$$

(2) (A): $\boxed{f(x,y) = 2xy\left(3 + \frac{x}{2} + \frac{y}{2}\right)}$

Determino i pti critici:

$$\begin{aligned} \partial_x f(x,y) &= \\ \partial_y f(x,y) &= \end{aligned} \begin{cases} 2y\left(3 + \frac{x}{2} + \frac{y}{2}\right) + xy = 0 \\ 2x\left(3 + \frac{x}{2} + \frac{y}{2}\right) + xy = 0 \end{cases}$$

$$\begin{cases} 2y\left(3 + x + \frac{y}{2}\right) = 0 \\ 2x\left(3 + \frac{x}{2} + y\right) = 0 \end{cases}$$

$$\begin{cases} y = 0 \\ x = 0 \end{cases} \Rightarrow \boxed{P_0 = (0,0)} \quad \vee \quad \begin{cases} y = -6 \\ x = 0 \end{cases} \Rightarrow \boxed{P_1 = (0,-6)}$$

$$\vee \begin{cases} y = 0 \\ x = -6 \end{cases} \Rightarrow \boxed{P_2 = (-6,0)} \quad \vee \quad \begin{cases} y = -6 - 2x \\ 3 + \frac{x}{2} - 6 - 2x = 0 \end{cases} \Rightarrow \begin{cases} \dots \\ -\frac{3}{2}x = 3 \end{cases} \Rightarrow \begin{cases} y = -2 \\ x = -2 \end{cases} \Rightarrow \boxed{P_3 = (-2,-2)}$$

CLASSIFICAZIONE:

$\partial_{xx} f(x,y) = 2y$

$\partial_{xy} f(x,y) = 6 + 2x + 2y$

$\partial_{yy} f(x,y) = 2x$

$\Rightarrow H_f(0,0) = \begin{pmatrix} 0 & 6 \\ 6 & 0 \end{pmatrix} \Rightarrow \det H_f < 0 \Rightarrow$ $P_0 = (0,0)$ è pto di SELLA

Analogamente: P_1 e P_2 pti di SELLA

Infine $H_f(-2,-2) = \begin{pmatrix} \overset{<0}{-4} & -2 \\ -2 & -4 \end{pmatrix}$

$\Rightarrow \det H_f(-2,-2) = 16 - 4 > 0 \Rightarrow$ P_3 è pto di MAX RELATIVO

(3) B $f(x,y) = 3xy(2+x+y)$

Determino i pti critici:

$$\begin{cases} \partial_x f(x,y) = 3y(2+x+y+x) = 0 \\ \partial_y f(x,y) = 3x(2+x+y+y) = 0 \end{cases}$$

$\rightarrow \begin{cases} y=0 \\ x=0 \end{cases}$ $\begin{cases} y=-2 \\ x=0 \end{cases}$ $\vee \begin{cases} y=0 \\ x=-2 \end{cases}$ $\vee \begin{cases} y=-2x-2 \\ 2+x+2(-2x-2)=0 \end{cases}$

$\boxed{P_0 = (0,0)}$ $\boxed{P_1 = (0,-2)}$ $\boxed{P_2 = (-2,0)}$

$\begin{cases} y=-2x-2 \\ -3x-2=0 \end{cases}$

$\begin{cases} y=-2/3 \\ x=-2/3 \end{cases}$

$\boxed{P_3 = (-2/3,-2/3)}$

Classificazione:

$\partial_{xx} f(x,y) = 6y$

$\partial_{xy} f(x,y) = 6+6x+6y$

$\partial_{yy} f(x,y) = 6x$

Procedendo come nell'altro esercizio si ottiene:

P_0, P_1, P_2 pti di SELLA

P_3 : pto di MAX relativo