1 (A) Determinare l'eq. del piano tangente al grafico di f nel punto P: (1)

$$f(x,y) = e^{x\cos y} \ln(y^2+2) \qquad P=(1,0)$$

$\boxed{f(1,0) = e\ln 2}$

$\partial_x f(x,y) = \cos y \, e^{x\cos y} \cdot \ln(y^2+2) \rightarrow \boxed{\partial_x f(1,0) = e\ln 2}$

$\partial_y f(x,y) = -x \,\mathrm{sen}\, y \, e^{x\cos y} \ln(y^2+2) + e^{x\cos y} \dfrac{2y}{y^2+2}$

$\rightarrow \boxed{\partial_y f(1,0) = \emptyset}$

$\Rightarrow$ EQ. PIANO TANGENTE :

$$\boxed{z = e\ln 2 + e\ln 2\,(x-1) = e\ln 2\, x}$$

(B) $f(x,y) = 2 + \arctan(y^2)\ln(y \,\mathrm{sen}\, x) \qquad P=\left(\frac{\pi}{2}, 1\right)$

$\boxed{f\left(\frac{\pi}{2}, 1\right) = 2}$

$\partial_x f(x,y) = \dfrac{\cancel{y}\cos x}{\cancel{y}\,\mathrm{sen}\, x} \cdot \arctan(y^2) \rightarrow \boxed{\partial_x f\left(\frac{\pi}{2}, 1\right) = \emptyset}$

$\partial_y f(x,y) = \dfrac{2y}{1+y^4} \ln(y\,\mathrm{sen}\, x) + \arctan(y^2) \dfrac{\cancel{\mathrm{sen}\, x}}{y\,\cancel{\mathrm{sen}\, x}}$

$\rightarrow \boxed{\partial_y f\left(\frac{\pi}{2}, 1\right) = \frac{\pi}{4}}$

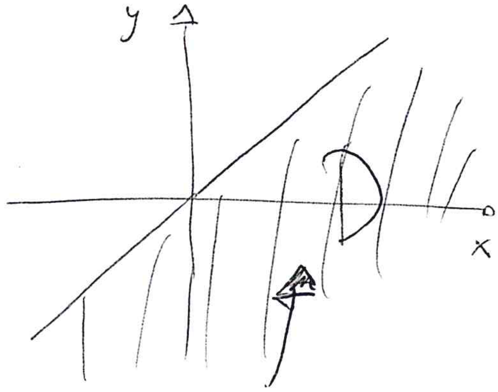
$\Rightarrow$ EQ. PIANO TANGENTE:

$$z = 2 + \frac{\pi}{4}(y-1) = \boxed{\frac{\pi}{4} y + 2 - \frac{\pi}{4}}$$

2) A Dominio e pti critici di

$$f(x,y) = (x+2)\ln(x-y)$$

DOMINIO: $x - y > 0 \Rightarrow x > y$



PTI CRITICI:

$$\begin{cases} \partial_x f(x,y) = \ln(x-y) + \dfrac{x+2}{x-y} = 0 \\ \partial_y f(x,y) : -\dfrac{(x+2)}{x-y} = 0 \end{cases} \Leftrightarrow \begin{cases} \ln(x-y) = 0 \\ x = -2 \end{cases}$$

$$\Rightarrow \begin{cases} -2-y = 1 \\ x = -2 \end{cases} \Leftrightarrow \begin{cases} y = -3 \\ x = -2 \end{cases}$$

C'è un solo pto critico $P = (-2,-3)$

Calcolo l'HESSIANA:

$$\partial_{xx} f(x,y) : \frac{1}{x-y} + \frac{\cancel{x} - y - \cancel{x} - 2}{(x-y)^2} \Rightarrow \partial_{xx} f(-2,-3) = 2$$

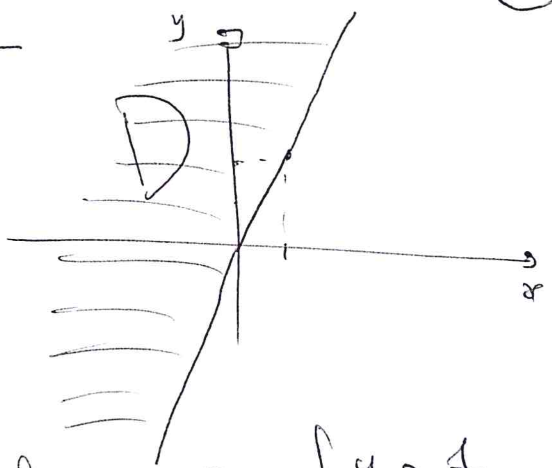
$$\partial_{xy} f(x,y) = -\frac{1}{x-y} + \frac{x+2}{(x-y)^2} \longrightarrow \partial_{xy} f(-2,-3) = -1$$

$$\partial_{yy} f(x,y) = -\frac{(x+2)}{(x-y)^2} \longrightarrow \partial_{yy} f(-2,-3) = 0$$

$$\Rightarrow H_f(-2,-3) = \begin{pmatrix} 2 & -1 \\ -1 & 0 \end{pmatrix} \Rightarrow \det H_f(-2,-3) = -1 < 0$$

$\Rightarrow$ P è pto di SELLA

2) B) $f(x,y) = x\ln(y-2x)$



DOMINIO: $y-2x>0 \iff y>2x$

PTI CRITICI:

$$\partial_x f(x,y): \quad \ln(y-2x) - \frac{2x}{y-2x} = 0$$

$$\partial_y f(x,y): \quad \frac{x}{y-2x} = 0 \rightarrow \begin{cases} \log y = 0 \\ x = 0 \end{cases} \rightarrow \begin{cases} y = 1 \\ x = 0 \end{cases}$$

Ho un solo pto critico: $P = (0,1)$

Calcolo l' HESSIANA:

$$\partial_{xx} f(x,y) = -\frac{2}{y-2x} - \left[\frac{2y - \cancel{4x} + \cancel{4x}}{(y-2x)^2}\right] \rightarrow \partial_{xx} f(0,1) = -2-2 = -4$$

$$\partial_{xy} f(x,y) = \frac{1}{y-2x} + \frac{2x}{(y-2x)^2} \rightarrow \partial_{xy} f(0,1) = 1$$

$$\partial_{yy} f(x,y) = -\frac{x}{(y-2x)^2} \longrightarrow \partial_{yy} f(0,1) = 0$$

$$\Rightarrow H_f(0,1) = \begin{pmatrix} -4 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow \det H_f(0,1) = -1 < 0$$

$\Rightarrow$ P è pto di SELLA