① (A) $\boxed{f(x,y) = \dfrac{\log(xy)}{\operatorname{sen}(x^2+y^2)}}$ $P=(1,1)$ $\vec{V}=(1,0)$ (1)

Calcolare la derivata direzionale di f lungo $\vec{V}$ nel pto P.

$$g(t) = f(P+t\vec{V}) = \frac{\log((1+t)\cdot 1)}{\operatorname{sen}((1+t)^2+1)} = \frac{\log(1+t)}{\operatorname{sen}((1+t)^2+1)}$$

$$g'(t) = \frac{1}{(1+t)\operatorname{sen}((1+t)^2+1)} - \frac{2(t+1)\cos((1+t)^2+1)\cdot\log(1+t)}{\operatorname{sen}((1+t)^2+1)}$$

$$\boxed{\Rightarrow \partial_V f(1,1) = g'(0) = \frac{1}{\operatorname{sen}(2)}}$$

Verifico se vale la formula del gradiente:

$$\partial_x f(x,y) = \frac{1\cancel{\operatorname{sen}(x^2+y^2)}}{x\operatorname{sen}(x^2+y^2)} - \frac{2x\cos(x^2+y^2)\cdot\log(xy)}{(\operatorname{sen}(x^2+y^2))^2}$$

$$\Rightarrow \partial_x f(1,1) = \frac{1}{\operatorname{sen}(2)}$$

$$\partial_y f(x,y) = \frac{1}{y\cdot\operatorname{sen}(x^2+y^2)} - \frac{2y\operatorname{sen}(x^2+y^2)\log(xy)}{(\operatorname{sen}(x^2+y^2))^2}$$

$$\Rightarrow \partial_y f(1,1) = \frac{1}{\operatorname{sen}(2)}$$

$$\Rightarrow \boxed{\partial_V f(1,1)} = \nabla f(1,1)\cdot V = \left(\frac{1}{\operatorname{sen}(2)}, \frac{1}{\operatorname{sen}(2)}\right)\cdot(1,0)$$

$$= \boxed{\frac{1}{\operatorname{sen}(2)}}$$

B. $f(x,y) = \dfrac{\log(x+y)}{\cos(x^2y^2)}$ $\quad P=(0,1) \quad V=(1,0)$ ②

$$g(t) = f(P+tV) = \frac{\log(t+1)}{\cos(t^2)}$$

$$\partial_V f(0,1) = g'(0)$$

$$g'(t) = \frac{1}{(t+1)\cdot\cos(t^2)} + \frac{2t\,\operatorname{sen}(t^2)\cdot\log(t+1)}{(\cos(t^2))^2}$$

$$\rightarrow \boxed{\partial_V f(0,1)} = g'(0) = \frac{1}{\cos(0)} + \emptyset = \boxed{1}$$

Verifico che vale la formula del gradiente:

$$\partial_x f(x,y) = \frac{1}{(x+y)\cos(x^2y^2)} + \frac{2xy^2\operatorname{sen}(x^2y^2)\log(x+y)}{\cos^2(x^2y^2)}$$

$$\rightarrow \partial_x f(0,1) = \frac{1}{\cos(0)} = 1$$

$$\partial_y f(x,y) = \frac{1}{(x+y)\cos(x^2y^2)} + \frac{2x^2y\operatorname{sen}(x^2y^2)\log(x+y)}{\cos^2(x^2y^2)}$$

$$\rightarrow \partial_y f(0,1) = 1$$

$$\boxed{\partial_V f(0,1)} = \nabla f(0,1)\cdot V = (1,1)\cdot(1,0) = \boxed{1}$$

$$\boxed{f(x,y) = 6x^2y^2 + 6y^4 - 3x^2 - y^2 + 18}$$



$$\begin{cases} 12xy^2 - 6x = 0 \\ 24y^3 + 12x^2y - 2y = 0 \end{cases} \qquad \begin{cases} 6x(2y^2-1) = 0 \\ \end{cases} \qquad \begin{cases} x = 0 \\ 2y(12y^2-1) = 0 \end{cases}$$

$P_0 = (0,0)$

$P_1 = \left(0, \frac{1}{\sqrt{12}}\right)$ $P_2 = \left(0, -\frac{1}{\sqrt{12}}\right)$ $\vee$ $\begin{cases} y = \pm 1 \\ \underbrace{24 + 12x^2 - 2 = 0}_{\text{NON HA SOLUZIONI}} \end{cases}$

$$H_f(x,y) = \begin{pmatrix} 12y^2 - 6 & 24xy \\ 24xy & 72y^2 + 12x^2 - 2 \end{pmatrix}$$

$$H_f(0,0) = \begin{pmatrix} -6 & 0 \\ 0 & -2 \end{pmatrix} < \emptyset$$

def negative

$(\det H_f(0,0) = 12)$

$P_0 = (0,0)$ MAX LOCALE

$$H_f\left(0, \pm\sqrt{\tfrac{1}{12}}\right) = \begin{pmatrix} -5 & 0 \\ 0 & 5 \end{pmatrix}$$

$\det H_f\left(0, \pm\frac{1}{\sqrt{12}}\right) < 0$

$\rightarrow$ $P_1$ e $P_2$ pti di sella

(B) $f(x,y) = 3x^2y^2 + 4y^4 - 3x^2 - y^2 + 9$



$$\begin{cases} 6xy^2 - 6x = 0 \\ 6x^2y + 16y^3 - 2y = 0 \end{cases} \qquad \begin{cases} 6x(y^2-1) = 0 \\ \end{cases} \qquad \begin{cases} x = 0 \\ 2y(8y^2-1) = 0 \end{cases} \cup \begin{cases} y = \pm 1 \\ 6x^2 + 16 \cdot ... \end{cases}$$

NON ha soluzioni

$\Rightarrow P_0 = (0,0) \quad P_1 = \left(0, \frac{1}{\sqrt{8}}\right) \quad P_2 = \left(0, -\frac{1}{\sqrt{8}}\right)$

$$H_f(x,y) = \begin{pmatrix} 6y^2 - 6 & 12xy \\ 12xy & 6x^2 + 48y^2 - 2 \end{pmatrix}$$

$$H_f(0,0) = \begin{pmatrix} -6 & 0 \\ 0 & -2 \end{pmatrix}$$

$\det H_f(0,0) = 12 > 0$ $\quad H_f(0,0)$ def negativa

$\rightarrow (0,0)$ pto di MAX locale

$$H_f\left(0, \pm\frac{1}{\sqrt{8}}\right) = \begin{pmatrix} \frac{3}{4} - 6 & 0 \\ 0 & 4 \end{pmatrix}$$

$\det H_f\left(0, \pm\frac{1}{\sqrt{8}}\right) < 0$

$P_1$ e $P_2$ pti di SELLA