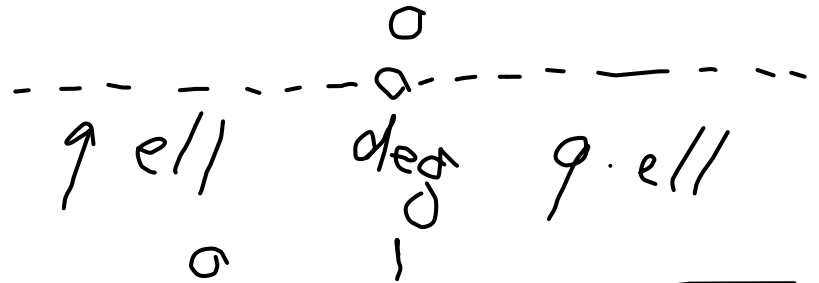


Esempio 2, es 1a Classificare $(k \in \mathbb{R})$

$$x^2 - 2xy + 2y^2 + 2(k-1)yz + 2yt + (1-k)z^2 - 2zt + t^2 = 0$$

$$A_k = \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & 2 & (k-1) \\ -1 & 0 & (k-1) & (1-k) \end{pmatrix}$$

$$|A_k| = -k^2$$

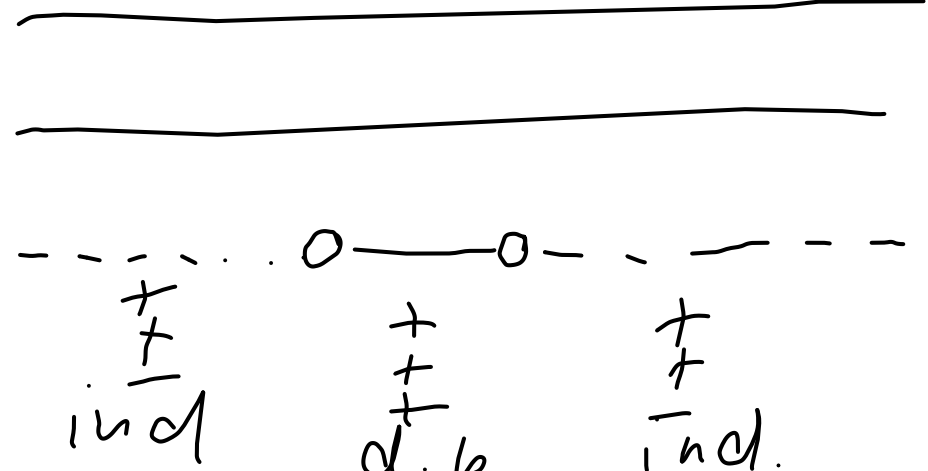


$$M_a^0 = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & (k-1) \\ 0 & (k-1) & (1-k) \end{pmatrix}$$

$$|M_1|$$

$$|M_2|$$

$$|M_3|$$



$$|M_1| = 1 > 0$$

$$|M_2| = 1 > 0$$

$$|M_3| = A_a^0 = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & (k-1) \\ 0 & (k-1) & (1-k) \end{vmatrix} = \begin{vmatrix} 1 & (k-1) \\ (k-1) & (1-k) \end{vmatrix} =$$

$$\begin{aligned} &= (1-k) - (k-1)^2 = \\ &= -k - k - k^2 + 2k - k = \\ &= -k(k-1) \end{aligned}$$

$$k=0 \quad k=2 \quad \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & 2 & -1 \\ -1 & 0 & -1 & 1 \end{pmatrix} \quad \begin{aligned} |M_1| &= \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix} = 0 \\ |M_2| &= \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ -1 & 0 & -1 \end{vmatrix} = 0 \end{aligned}$$

| k | $ A $ | $\text{rk} A$ | M_0 | Quadriche |
|-------------|-------|---------------|------------|-----------------------|
| $k < 0$ | - | 4 | quad. ind. | iperboloidi ellittici |
| $k = 0$ | 0 | 2 | 0 | deg $\text{rk} = 2$ |
| $0 < k < 1$ | - | 4 | quad. d.p. | ellissoidi reali |
| $k = 1$ | - | 4 | quad. deg | paraboloidi ellittico |
| $k > 1$ | - | 4 | ind. | iperboloidi ellittici |

$$Q_\infty = \text{Im}[f] \cap \Pi_\infty$$

$$[f] \quad A = \begin{pmatrix} a_0^0 & \dots & a_n^0 \\ a_0^1 & \dots & a_n^1 \\ \vdots & & \vdots \\ a_0^n & \dots & a_n^n \end{pmatrix}$$

Cerco i poli di iperpiani principali. Considero il generico punto improprio $P_\infty \equiv (0, l_1, \dots, l_n)$. Ne faccio

l'iperpiano polare $\pi(P_\infty): (X_0, X_1, \dots, X_n) A \begin{pmatrix} 0 \\ l_1 \\ \vdots \\ l_n \end{pmatrix} = 0$

$$(X_0, X_1, \dots, X_n) \begin{pmatrix} a_0^0 \cdot 0 + a_1^0 l_1 + \dots + a_n^0 l_n \\ a_0^1 \cdot 0 + a_1^1 l_1 + \dots + a_n^1 l_n \\ \vdots \\ a_0^n \cdot 0 + a_1^n l_1 + \dots + a_n^n l_n \end{pmatrix} \begin{matrix} b_0 \\ b_1 \\ \vdots \\ b_n \end{matrix} = 0$$

$$b_0 X_0 + b_1 X_1 + \dots + b_n X_n = 0$$

$$b_0 + b_1 x_1 + \dots + b_n x_n = 0$$

condizione di $\perp_{\mathbb{S}^n}$
 $(l_1, \dots, l_n) \sim (b_1, \dots, b_n)$

Quindi deve esistere $\lambda \neq 0$ tale che

$$\begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = \lambda \cdot \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix} \quad M \Rightarrow$$

$$\begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = M_0^0 \cdot \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}$$

Perciò deve essere

$$\exists \lambda \neq 0 \text{ t.c.} \quad M_0^0 \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix} = \lambda \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}$$

a) Classificare la conica $\Gamma: x^2 - 6xy - 7y^2 + 4x - 16 = 0$
 (verificare che è un'iperbole)

b) Trovarne gli assi c) Trovarne gli asintoti d) Trovarne il centro

$$A = \begin{pmatrix} -1 & 2 & 0 \\ 2 & 1 & -3 \\ 0 & -3 & -7 \end{pmatrix} \quad |A| = \begin{vmatrix} -1 & 2 & 0 \\ 0 & 5 & -3 \\ 0 & -3 & -7 \end{vmatrix} = - \begin{vmatrix} 5 & -3 \\ -3 & -7 \end{vmatrix} = 44 \neq 0 \text{ non deg}$$

$$M_{00} = \begin{vmatrix} 1 & -3 \\ -3 & -7 \end{vmatrix} = -16 < 0 \text{ iperbole}$$

b) $\begin{vmatrix} \lambda - 1 & 3 \\ 3 & \lambda + 7 \end{vmatrix} = \lambda^2 - \lambda + 7\lambda - 7 - 9 = \lambda^2 + 6\lambda - 16 \quad \lambda = -3 \pm \sqrt{9 + 16} = \begin{matrix} 2 \\ -8 \end{matrix}$

$U_2: \begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix} \begin{pmatrix} l \\ m \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad l + 3m = 0 \quad l = -3m \quad \text{scelgo } (-3, 1)$

$U_{-8}: \begin{pmatrix} -9 & 3 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} l \\ m \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad 3l - m = 0 \quad m = 3l$
 scelgo $(1, 3)$

$(0 \ 13) \cdot A \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 0 \quad \boxed{2 - 8x - 34y = 0}$

$(0 \ -3 \ 1) \cdot A \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 0 \quad \boxed{-6 - 6x + 2y = 0}$

$$x^2 - 6xy - 7y^2 + 4x - t^2 = 0$$

$$\left. \begin{array}{l} t=0 \\ t=0 \end{array} \right\} \begin{array}{l} x^2 - 6xy - 7y^2 = 0 \\ x^2 - 6xy - 7y^2 = 0 \end{array}$$

$$A = \begin{pmatrix} -1 & 2 & 0 \\ 2 & 1 & -3 \\ 0 & -3 & -7 \end{pmatrix}$$

M₀

$$\left. \begin{array}{l} x = 3y \pm \sqrt{9y^2 + 7y^2} = y(3 \pm 4) \\ t = 0 \end{array} \right\} \begin{array}{l} 7y \\ -y \end{array}$$

$$P_{1\infty} \equiv (0, 7, 1)$$

$$\mathcal{P}(P_{1\infty}) : (0 \ 7 \ 1) \cdot A \cdot \begin{pmatrix} 1 \\ x \\ y \end{pmatrix} = 0$$

$$16 + 4x - 24y = 0$$

$$P_{2\infty} \equiv (0, -1, 1) \quad (0 \ -1 \ 1) \cdot A \cdot \begin{pmatrix} 1 \\ x \\ y \end{pmatrix} = 0$$

d) Centro: o inters dei due assi o inters dei due
 & sintoti, o intersez, di 2 diametri qualsiasi;