Esempio 2, es 1a Classificare ($k \in \mathbb{R}$)

$$x^2 - 2xy + 2y^2 + 2(k-1)yz + 2yt + (1-k)z^2 - 2zt + t^2 = 0$$

$$A_k = \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & 2 & (k-1) \\ -1 & 0 & (k-1) & (1-k) \end{pmatrix} \qquad |A_k| = -k^2$$

$$M_a^0 = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & (k-1) \\ 0 & (k-1) & (1-k) \end{pmatrix}$$

$|M_1| = 1 > 0$

$|M_2| = 1 > 0$

$$|M_3| = A_0^0 = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & (k-1) \\ 0 & (k-1) & (1-k) \end{vmatrix} = \begin{vmatrix} 1 & (k-1) \\ (k-1) & (1-k) \end{vmatrix} = (1-k) - (k-1)^2 = \cancel{1} - k - k^2 + 2k - \cancel{1} = -k(k-1)$$

		0		1	
$\lvert A_k \rvert$	$-$	0	$-$		$-$
	q ell	deg	q ell		
$\lvert M_1 \rvert$	$+$		$+$		$+$
$\lvert M_2 \rvert$	$+$		$+$		$+$
$\lvert M_3 \rvert$	$-$	0	$+$	0	$-$
	ind		d.p.		ind

$$k = 0 \quad \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & 2 & -1 \\ -1 & 0 & -1 & 1 \end{pmatrix} \qquad |M_1| = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{vmatrix} = 0 \qquad |M_2| = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & -1 \end{vmatrix} = 0$$

$rk = 2$

k	$\lvert A \rvert$	$rk A$	M^0_0	Quadriche
$k<0$	$-$	4	ind.	iperboloidi ellittici
$k=0$	0	2	0	deg $rk=2$
$0<k<1$	$-$	4	d.p.	ellissoidi reali
$k=1$	$-$	4	0 deg	paraboloide ellittico
$k>1$	$-$	4	ind.	iperboloidi ellittici

$$Q_\infty = \mathrm{Im}[f] \cap \Pi_\infty$$

[f] $A = \begin{pmatrix} a_0^0 & \cdots & a_n^0 \\ a_0^1 & \cdots & a_n^1 \\ \cdots & \cdots & \cdots \\ a_0^n & \cdots & a_n^n \end{pmatrix}$ Cerco i poli di iperpiani principali. Considero il generico punto improprio $P_\infty \equiv (0, l_1, \dots, l_n)$. Ne faccio l'iperpiano polare $\tau(P_\infty)$: $(X_0 X_1 \cdots X_n) A \begin{pmatrix} 0 \\ l_1 \\ \vdots \\ l_n \end{pmatrix} = 0$

$$(X_0 X_1 \cdots X_n) \cdot \begin{pmatrix} a_0^0 0 + a_1^0 l_1 + \cdots + a_n^0 l_n \\ a_0^1 0 + a_1^1 l_1 + \cdots + a_n^1 l_n \\ \cdots \\ a_0^n 0 + a_1^n l_1 + \cdots + a_n^n l_n \end{pmatrix} \begin{matrix} b_0 \\ b_1 \\ \\ b_n \end{matrix} = 0$$

$b_0 X_0 + b_1 X_1 + \cdots + b_n X_n = 0$

$b_0 + b_1 x_1 + \cdots + b_n x_n = 0$

condizione di $\perp$:

$(l_1, \dots, l_n) \sim (b_1, \dots, b_n)$

Quindi deve esistere $\lambda \neq 0$ tale che

$$\begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = \lambda \cdot \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix} \cdot M_\lambda$$

$$\begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = M_0^0 \cdot \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}$$

Perciò deve essere:

$$\exists \lambda \neq 0 \text{ t.c. } M_0^0 \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix} = \lambda \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}$$

a) Classificare la conica $\Gamma: x^2-6xy-7y^2+4xt-t^2=0$ (verificare che è un'iperbole)
b) Trovarne gli assi c) Trovarne gli asintoti d) Trovarne il centro

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a) $A=\begin{pmatrix}-1 & 2 & 0\\ 2 & \boxed{\begin{matrix}1 & -3\\ -3 & -7\end{matrix}}\end{pmatrix}$ $M^0_0=$ $\quad |A|=\begin{vmatrix}-1 & 2 & 0\\ 0 & 5 & -3\\ 0 & -3 & -7\end{vmatrix}=-\begin{vmatrix}5 & -3\\ -3 & -7\end{vmatrix}=44\neq 0$ non deg

$A^0_0=\begin{vmatrix}1 & -3\\ -3 & -7\end{vmatrix}=-16<0$ iperbole

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b) $\begin{vmatrix}(\lambda-1) & 3\\ 3 & (\lambda+7)\end{vmatrix}=\lambda^2-\lambda+7\lambda-7-9=\lambda^2+6\lambda-16 \quad \lambda=-3\pm\sqrt{9+16}=\begin{cases}2\\-8\end{cases}$

$U_2: \begin{pmatrix}1 & 3\\ \cancel{3} & \cancel{9}\end{pmatrix}\begin{pmatrix}l\\ m\end{pmatrix}=\begin{pmatrix}0\\ \cancel{0}\end{pmatrix}$ $\quad l+3m=0 \quad l=-3m$ scelgo $(-3,1)$

$(0\ -3\ 1)\cdot A\cdot\begin{pmatrix}1\\ x\\ y\end{pmatrix}=0$

$\boxed{-6-6x+2y=0}$

$U_{-8}: \begin{pmatrix}\cancel{-9} & \cancel{3}\\ 3 & -1\end{pmatrix}\begin{pmatrix}l\\ m\end{pmatrix}=\begin{pmatrix}\cancel{0}\\ 0\end{pmatrix}$ $\quad 3l-m=0 \quad m=3l$

scelgo $(1,3)$ $\quad (0\ 1\ 3)\cdot A\cdot\begin{pmatrix}1\\ x\\ y\end{pmatrix}=0$ $\boxed{2-8x-24y=0}$

$$\Gamma: x^2 - 6xy - 7y^2 + 4xt - t^2 = 0$$

$$\begin{cases} \ldots \\ t = 0 \end{cases} \quad \begin{cases} x^2 - 6xy - 7y^2 = 0 \\ t = 0 \end{cases}$$

$$A = \begin{pmatrix} -1 & 2 & 0 \\ 2 & 1 & -3 \\ 0 & -3 & -7 \end{pmatrix} \quad M^0 =$$

$$\begin{cases} x = 3y \pm \sqrt{9y^2 + 7y^2} = y(3 \pm 4) = \begin{cases} 7y \\ -y \end{cases} \\ t = 0 \end{cases}$$

$$P_{1\infty} \equiv (0, 7, 1)$$

$$r(P_{1\infty}): (0\ 7\ 1) A \cdot \begin{pmatrix} 1 \\ x \\ y \end{pmatrix} = 0 \quad \boxed{14 + 4x - 24y = 0}$$

$$P_{2\infty} \equiv (0, -1, 1) \quad (0\ -1\ 1) \cdot A \begin{pmatrix} 1 \\ x \\ y \end{pmatrix} = 0 \quad \rule{3cm}{0.4pt}$$

d) Centro: o inters. dei due assi o inters. dei due asintoti o intersez. di 2 diametri qualsiasi