Verificare che $\Gamma: x^2-6xy+9y^2-2y+1=0$ è una parabola e trovarne l'asse.

$$A=\begin{pmatrix}1&0&-1\\0&1&-3\\-1&-3&9\end{pmatrix}\qquad |A|=\begin{vmatrix}1&0&-1\\0&1&-3\\0&-3&8\end{vmatrix}=\begin{vmatrix}1&-3\\-3&8\end{vmatrix}=-1\neq 0 \text{ non deg.}$$

M^0_α

$$A^0_0=|M^0_0|=9-9=0 \text{ parabola}$$

$$\begin{vmatrix}(\lambda-1)&3\\3&(\lambda-9)\end{vmatrix}=\lambda^2-\lambda-9\lambda+9-9=$$
$$=\lambda^2-10\lambda=\lambda(\lambda-10)$$

autoval: $0, 10$

$\lambda=10$

$$U_{10}:\begin{pmatrix}\cancel{9\;\;3}\\3\;\;1\end{pmatrix}\begin{pmatrix}l\\m\end{pmatrix}=\begin{pmatrix}\cancel{0}\\0\end{pmatrix}\qquad 3l+m=0 \qquad m=-3l \qquad (1,-3)$$

$$(0\;1\;-3)\cdot A\cdot\begin{pmatrix}1\\x\\y\end{pmatrix}=0$$

$$\boxed{3+10x-30y=0}$$

NON FATE QUESTO!

$$\lambda=0\quad\begin{pmatrix}-1&3\\\cancel{3\;\;-9}\end{pmatrix}\begin{pmatrix}l\\m\end{pmatrix}=\begin{pmatrix}0\\\cancel{0}\end{pmatrix}$$

$$-l+3m=0$$

$$(l,m)\sim(3,1)$$

$$(0\;3\;1)\cdot A\cdot\begin{pmatrix}1\\x\\y\end{pmatrix}\begin{matrix}X_0\\X_1\\X_2\end{matrix}=0$$

$$-1+0x+0y=0\qquad -1=0\;!$$

$$-X_0+0X_1+0X_2=0\qquad -X_0=0$$

Trovare i piani principali di

$$Q: x^2+y^2+9z^2-4xy+z=0$$

$$A=\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 9 \end{pmatrix}$$

M_0

$$\begin{vmatrix} \lambda-1 & 2 & 0 \\ 2 & \lambda-1 & 0 \\ 0 & 0 & \lambda-9 \end{vmatrix} = (\lambda-9)\left((\lambda-1)^2-2^2\right) = (\lambda-9)(\lambda-3)(\lambda+1)$$

autoval	ma	mg
-1	1	1
3	1	1
9	1	1

$$U_{-1}: \begin{pmatrix} -2 & 2 & 0 \\ \cancel{2} & \cancel{-2} & \cancel{0} \\ 0 & 0 & -10 \end{pmatrix}\begin{pmatrix} l \\ m \\ n \end{pmatrix}=\begin{pmatrix} 0 \\ \cancel{0} \\ 0 \end{pmatrix} \quad \begin{cases} -2l+2m=0 \\ -10n=0 \end{cases} \quad \lambda(1,1,0)$$

$$(0\ 1\ 1\ 0)\cdot A\cdot\begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix}=0$$

$$U_3: \begin{pmatrix} \cancel{2} & \cancel{2} & \cancel{0} \\ 2 & 2 & 0 \\ 0 & 0 & -6 \end{pmatrix}\begin{pmatrix} l \\ m \\ n \end{pmatrix}=\begin{pmatrix} \cancel{0} \\ 0 \\ 0 \end{pmatrix} \quad \begin{cases} 2l+2m=0 \\ -6n=0 \end{cases} \quad \lambda(1,-1,0)$$

$$(0\ 1\ -1\ 0)\cdot A\cdot\begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix}=0$$

$$U_9: \begin{pmatrix} 8 & 2 & 0 \\ 2 & 8 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix}\begin{pmatrix} l \\ m \\ n \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ \cancel{0} \end{pmatrix} \quad \begin{cases} 8l+2m=0 \\ 2l+8m=0 \end{cases} \quad \lambda(0,0,1)$$

$$(0\ 0\ 0\ 1)\cdot A\cdot\begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix}=0$$

$$9z=0$$

Trovare i piani principali di $Q: x^2+y^2+3z^2-1=0$

$$A=\begin{pmatrix}-1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&3\end{pmatrix}\qquad M^{\alpha}_{0}$$

$$\begin{vmatrix}(\lambda-1)&0&0\\0&(\lambda-1)&0\\0&0&(\lambda-3)\end{vmatrix}=(\lambda-1)^2(\lambda-3)$$

autoval.	ma	mg
1	2	2
3	1	1

$$U_3:\ \begin{pmatrix}2&0&0\\0&2&0\\\cancel{0}&\cancel{0}&\cancel{0}\end{pmatrix}\begin{pmatrix}l\\m\\n\end{pmatrix}=\begin{pmatrix}0\\0\\\cancel{0}\end{pmatrix}\qquad\begin{cases}2l=0\\2m=0\end{cases}\qquad (l,m,n)\sim(0,0,1)$$

$$(0\ 0\ 0\ 1)\cdot A\cdot\begin{pmatrix}1\\x\\y\\z\end{pmatrix}=0\qquad\boxed{3z=0}$$

$$U_1:\ \begin{pmatrix}\cancel{0}&\cancel{0}&\cancel{0}\\\cancel{0}&\cancel{0}&\cancel{0}\\0&0&-2\end{pmatrix}\begin{pmatrix}l\\m\\n\end{pmatrix}=\begin{pmatrix}\cancel{0}\\\cancel{0}\\0\end{pmatrix}\qquad -2n=0\qquad (l,m,n)=(\alpha,\beta,0)$$

$$(0\ \alpha\ \beta\ 0)\cdot A\cdot\begin{pmatrix}1\\x\\y\\z\end{pmatrix}=0$$

$$\alpha x+\beta y=0\qquad(\alpha,\beta)=(1,0)$$

Fascio di piani di asse $\begin{cases}x=0\\y=0\end{cases}$ $\quad(\alpha,\beta)=(0,1)$

$(\alpha,\beta)=(1,2)\quad x+2y=0$

$=(3,6)\quad 3x+6y=0$

Verificare che la quadrica Q,
$$3x^2+2y^2+2z^2+2yz+6y+6z-1=0$$
è di rotazione e trovarne il sottospazio di rotazione

$$A=\begin{pmatrix} -1 & 0 & 3 & 3 \\ 0 & 3 & 0 & 0 \\ 3 & 0 & 2 & 1 \\ 3 & 0 & 1 & 2 \end{pmatrix}$$

$$\begin{vmatrix} (\lambda-3) & 0 & 0 \\ 0 & (\lambda-2) & -1 \\ 0 & -1 & (\lambda-2) \end{vmatrix} = (\lambda-3)\left((\lambda-2)^2-(-1)^2\right) = (\lambda-3)(\lambda-3)(\lambda-1)$$

autoval	ma	mg	
3	2	2	rot.
1	1	1	

$$U_3:\ \begin{pmatrix} \cancel{0} & \cancel{0} & \cancel{0} \\ 0 & 1 & -1 \\ \cancel{0} & \cancel{-1} & \cancel{1} \end{pmatrix}\begin{pmatrix} l \\ m \\ n \end{pmatrix} = \begin{pmatrix} \cancel{0} \\ 0 \\ \cancel{0} \end{pmatrix}$$

$m-n=0$

$(l,m,n)\in\{(\alpha,\beta,\beta)\mid \alpha,\beta\in\mathbb{R}\}$

Base: $\{(1,0,0),(0,1,1)\}$

asse di rot.
$$\begin{cases}(0\,1\,0\,0)\cdot A\cdot\begin{pmatrix}1\\x\\y\\z\end{pmatrix}=0\\ (0\,0\,1\,1)\cdot A\cdot\begin{pmatrix}1\\x\\y\\z\end{pmatrix}=0\end{cases}$$

asse di rot.
$$\begin{cases} 3x=0 \\ 6+3y+3z=0 \end{cases}$$

(ogni $3\alpha x+\beta(6+3y+3z)=0$ è piano principale) ell. re.

$$\begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 2 & \\ & & & 3 \end{pmatrix} \quad \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 2 & \\ & & & 2 \end{pmatrix} \quad \begin{pmatrix} -1 & & & \\ & 2 & & \\ & & 2 & \\ & & & 2 \end{pmatrix}$$

iperboloide

$$\begin{pmatrix} -1 & & & \\ & 1 & & \\ & & -2 & \\ & & & -3 \end{pmatrix} \quad \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & -2 & \\ & & & -2 \end{pmatrix}$$

paraboloide ell.

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 2 & & \\ 0 & & 3 & \\ 1 & & & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 2 & & \\ 0 & & 2 & \\ 1 & & & 0 \end{pmatrix}$$

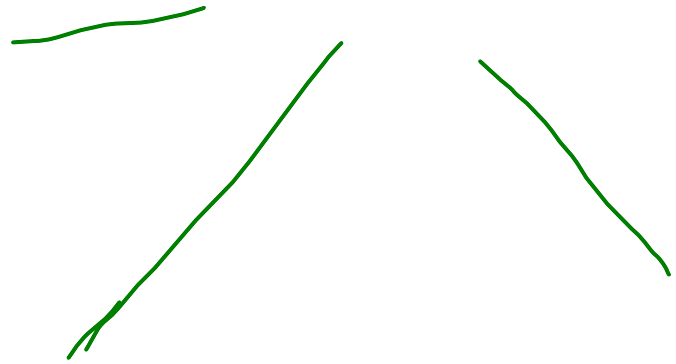
paraboloide ip.

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 2 & & \\ 0 & & -3 & \\ 1 & & & 0 \end{pmatrix}$$

Conica impropria di un ellissoide

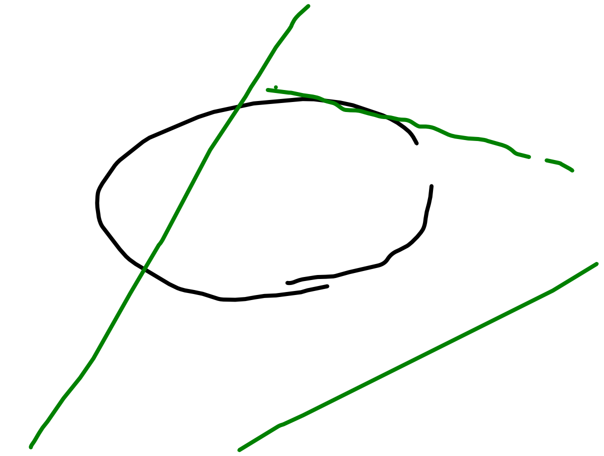
(∅)

Retta impr. di un piano

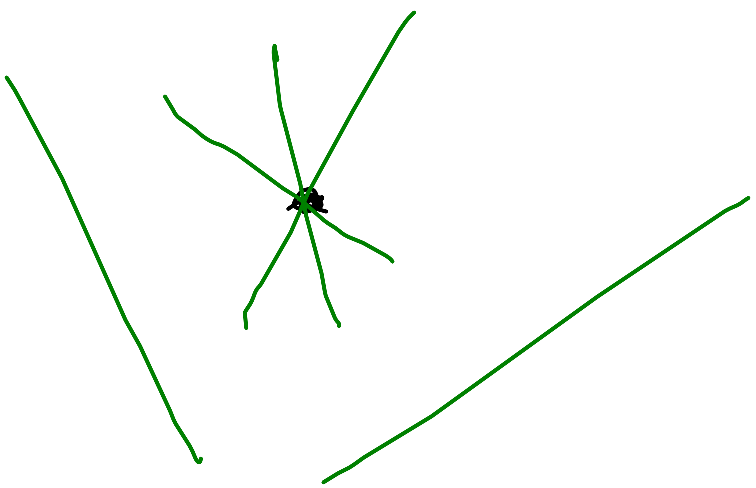


Conica impropria di un iperboloide

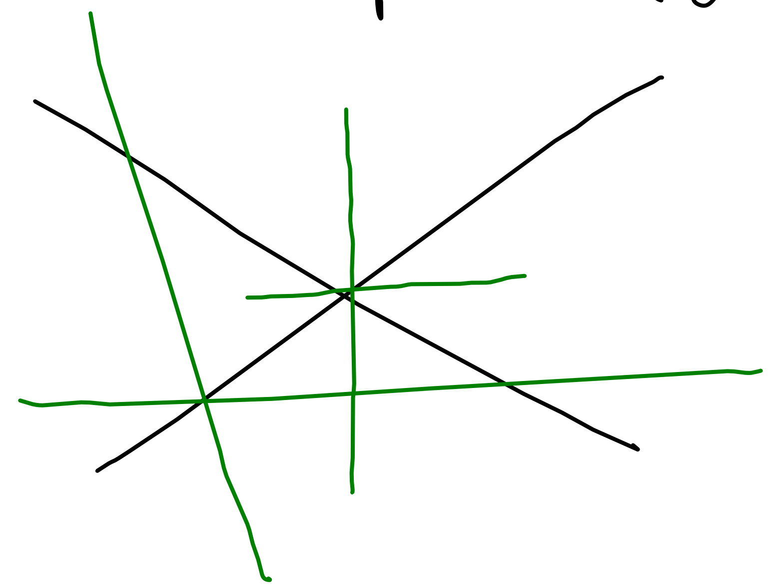
R. impr di un piano

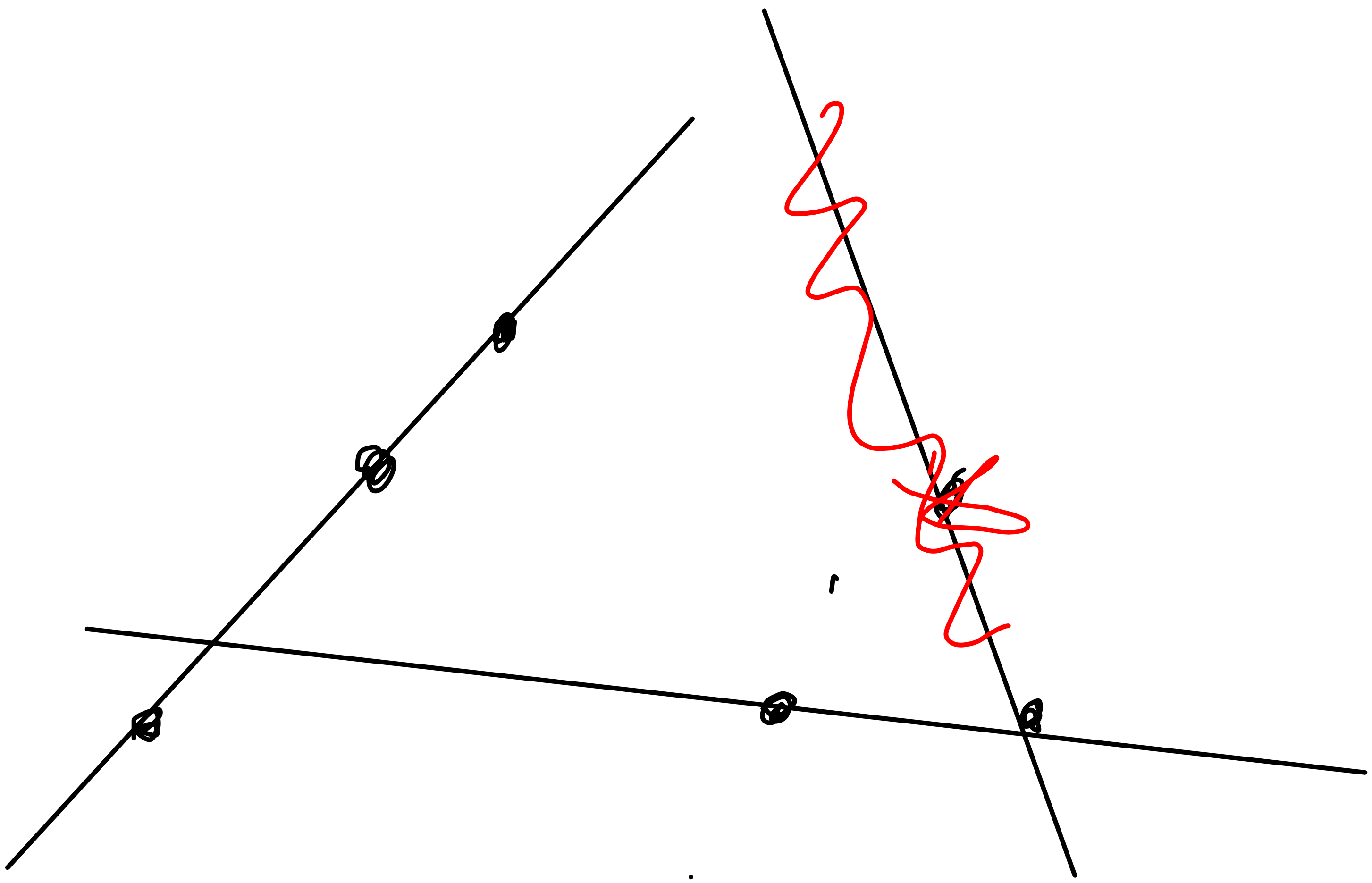


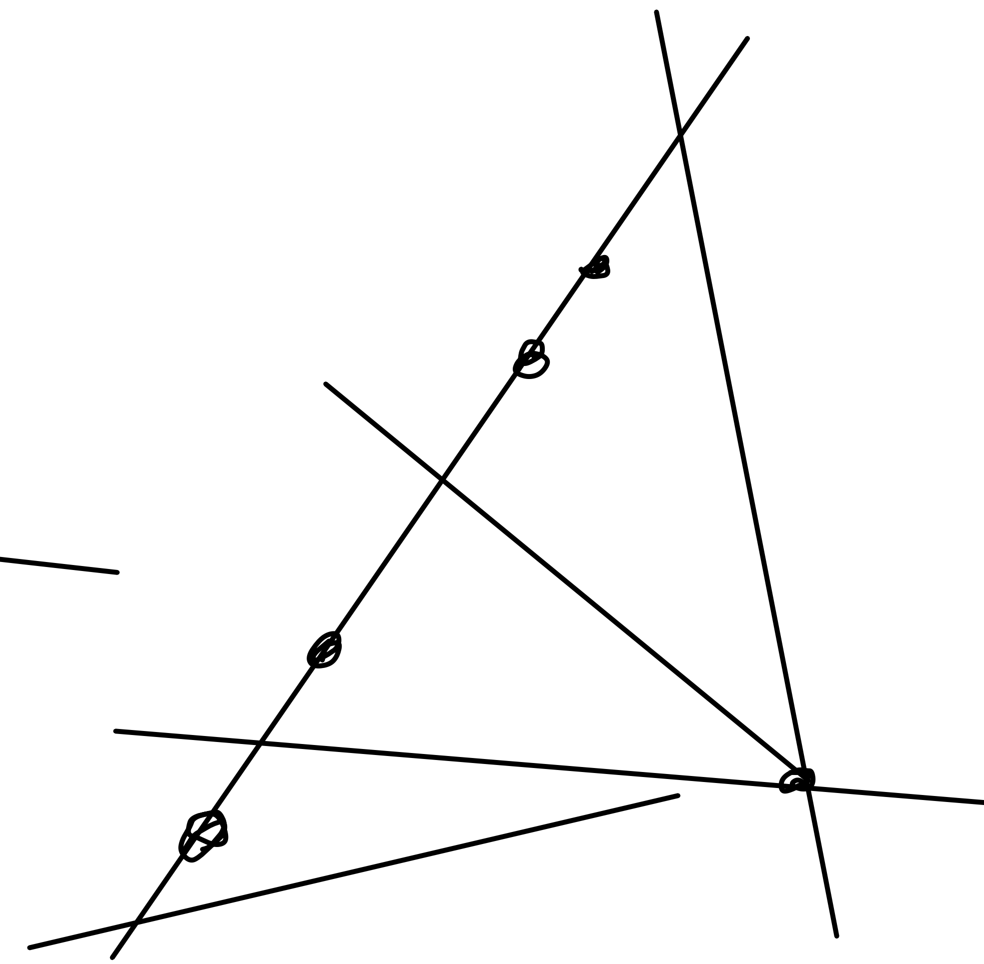
Conica impropria di un paraboloide ell.



Conica impropria di un paraboloide iperbolico







$[f]$ $[g]$

A B

$P \in \mathrm{Im}[f]$ $P \in \mathrm{Im}[g]$ $P = \begin{pmatrix} \bar{X}_0 \\ \vdots \\ \bar{X}_n \end{pmatrix}$

${}^t(\bar{X}) \cdot A \cdot (\bar{X}) = 0$ ${}^t(\bar{X}) \cdot B \cdot (\bar{X}) = 0$

Una conica del fascio $[h]$ Q

$[h] = [\lambda f + \mu g]$

$\lambda A + \mu B$

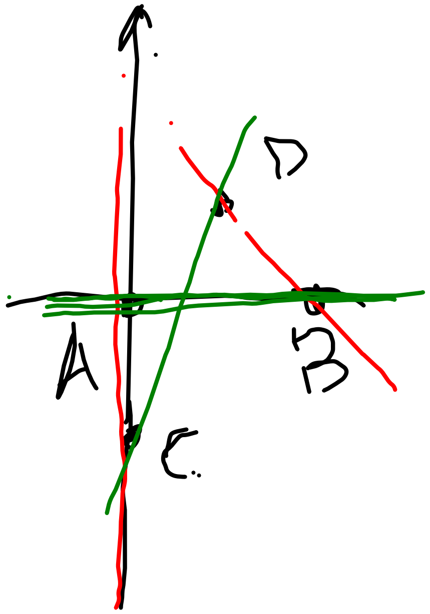
$P \in \mathrm{Im}[h]$

$${}^t(\bar{X}) \cdot (\lambda A + \mu B) \cdot (\bar{X}) =$$

$$= \lambda\, {}^t(\bar{X}) \cdot A \cdot (\bar{X}) + \mu\, {}^t(\bar{X}) \cdot B \cdot (\bar{X}) =$$

$$= \lambda \cdot 0 + \mu \cdot 0 = 0$$

Sì



$A \equiv (0,0)$, $B \equiv (3,0)$, $C \equiv (0,-1)$ $D \equiv (1,1)$

Trovare il fascio $\mathcal{F}$ per A, B, C, D

$\Gamma_1 = AC \cup BD$

$AC: x = 0$ $BD: \dfrac{x-3}{1-3} = \dfrac{y-0}{1-0}$

$\dfrac{x-3}{-2} = y$

$x + 2y - 3 = 0$

$\Gamma_1: x \cdot (x+2y-3) = 0$

$\Gamma_2 = AB \cup CD$

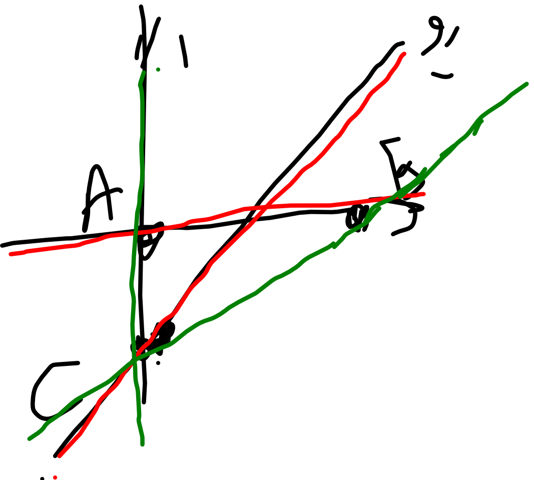
$AB: y = 0$ $CD: \dfrac{x-0}{1-0} = \dfrac{y+1}{1+1}$

$x = \dfrac{y+1}{2}$

$2x - y - 1 = 0$

$\Gamma_2: y(2x-y-1) = 0$

$$\boxed{\mathcal{F}: \lambda x \cdot (x+2y-3) + \mu\, y(2x-y-1) = 0}$$



$A \equiv (0,0) \quad B \equiv (3,0) \quad C \equiv (0,-1) \quad r: x-y-1=0$

Trovare il fascio $\mathcal{F}$ di coniche per A, B e tangenti in C ad r

$AB: y=0$

$\Gamma_1 = r \cup AB$

$\Gamma_1: y\cdot(x-y-1)=0$

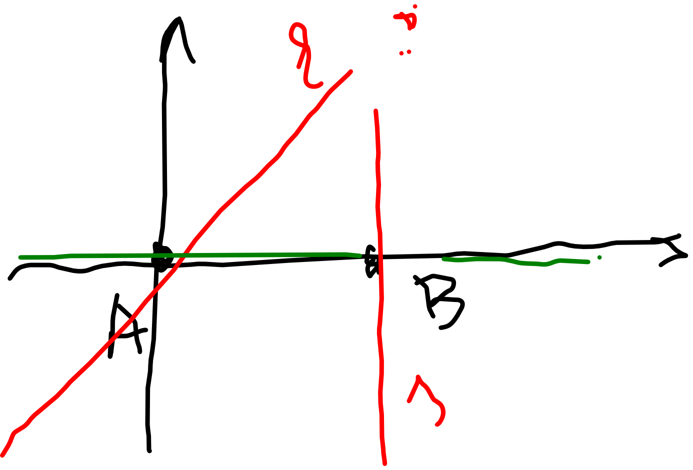
$AC: x=0 \qquad BC: \dfrac{x-0}{3-0} = \dfrac{y+1}{1}$

$\Gamma_2 = AC \cup BC$

$\Gamma_2: x\cdot(x-3y-3)=0 \qquad \dfrac{x}{3} = y+1$

$x-3y-3=0$

$\mathcal{F}: \lambda\, y(x-y-1) + \mu\, x(x-3y-3) = 0$



$A = (0,0)$ $B = (3,0)$ $r: x - y = 0$

$s: x - 3 = 0$

Trovare il fascio $\mathcal{F}$ di coniche tangenti in A ad r e in B ad s.

$\Gamma_1 = r \cup s$ $\Gamma_1 : (x-y)(x-3) = 0$

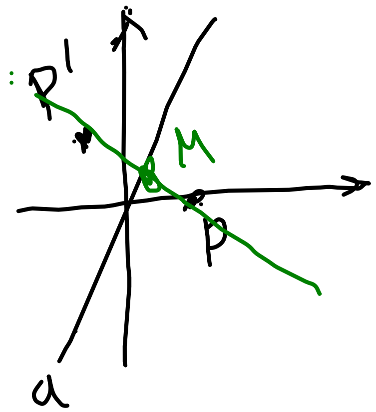
$\Gamma_2 =$ AB contata 2 volte $\Gamma_2 : y^2 = 0$

$$\mathcal{F} : \lambda (x-y)(x-3) + \mu y^2 = 0$$

Sia $a: y - 2x = 0$ $\quad P \equiv (1,0)$

$2x - y = 0$

Trovare il fascio $\mathcal{F}$ di parabole aventi a come asse e passanti per P.



Cerca P' simmetrico di P risp. ad a

Retta per $P \perp a$ $\quad \dfrac{x-1}{2} = \dfrac{y-0}{-1}$ $\quad -x+1 = 2y$

$x + 2y - 1 = 0$

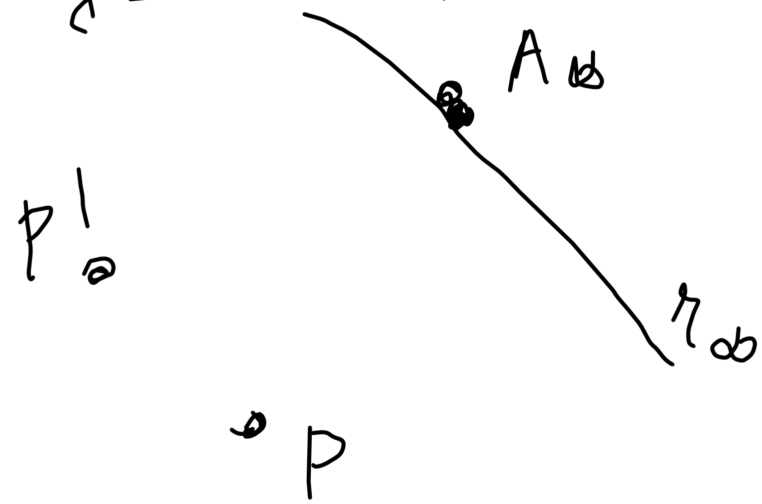
$$M: \begin{cases} 2x - y = 0 \\ x + 2y - 1 = 0 \end{cases}$$

$$\begin{cases} x + 2y - 1 = 0 \\ -5y + 2 = 0 \end{cases} \quad \begin{cases} x = 1 - \frac{4}{5} = \frac{1}{5} \\ y = \frac{2}{5} \end{cases} \quad M \equiv \left(\frac{1}{5}, \frac{2}{5}\right)$$

$$\frac{P + P'}{2} = M \qquad P' = 2M - P \equiv \left(\frac{2}{5}, \frac{4}{5}\right) - (1,0) = \left(-\frac{3}{5}, \frac{4}{5}\right)$$

Tutte le parabole di $\mathcal{F}$ sono tangenti alla retta impropria r_∞ nel punto improprio A_∞ dell'asse a.

$a: 2x - y = 0$ coeff. dir. $(l, m) \sim (1, 2)$ $A_\infty \equiv (0, 1, 2)$



$\Gamma_1 = r_\infty \cup PP'$ $r_\infty: X_0 = 0$ $PP': X_1 + 2X_2 - X_0 = 0$

$\Gamma_2 = PA_\infty \cup P'A_\infty$

$\Gamma_1: X_0(X_1 + 2X_2 - X_0) = 0$

$PA_\infty: \dfrac{x-1}{1} = \dfrac{y-0}{2}$ $2x - 1 - y = 0$

$2X_1 - X_0 - X_2 = 0$

$P'A_\infty: \dfrac{x+\frac{3}{5}}{1} = \dfrac{y-\frac{4}{5}}{2}$ $2x + \frac{6}{5} - y + \frac{4}{5} = 0$

$2x - y + 2 = 0$

$2X_1 - X_2 + 2X_0 = 0$

$\Gamma_2: (2X_1 - X_0 - X_2)(2X_1 - X_2 + 2X_0) = 0$

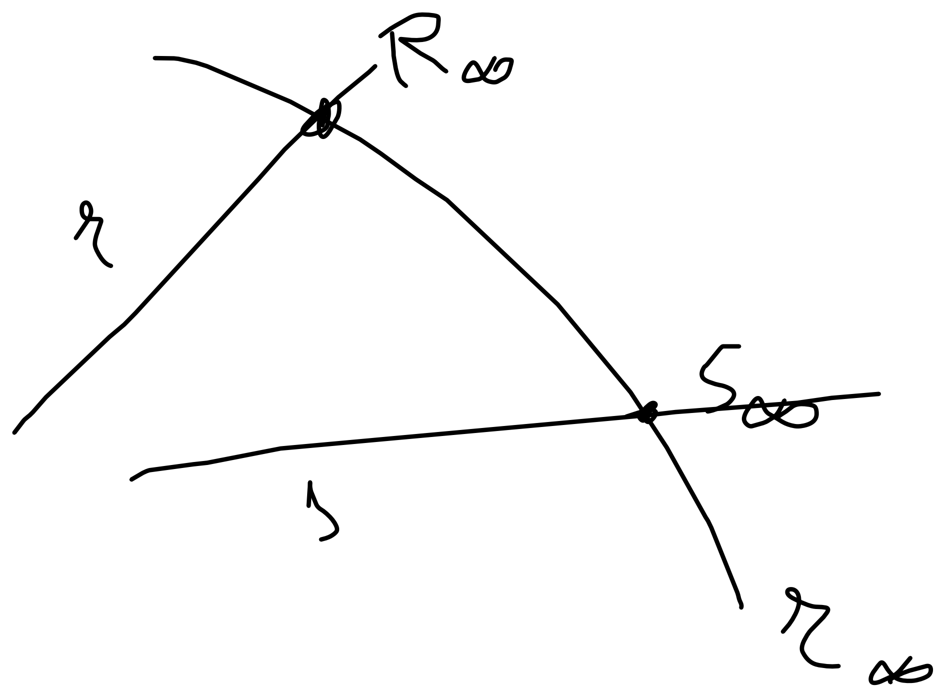
$\mathcal{F}: \lambda X_0(X_1 + 2X_2 - X_0) + \mu(2X_1 - X_0 - X_2)(2X_1 - X_2 + 2X_0) = 0$

$\mathcal{F}: \lambda(x + 2y - 1) + \mu(2x - 1 - y)(2x - y + 2) = 0$

Trovare il fascio di iperboli aventi asintoti

$r: x - y = 0$, $s: x + 2y - 1 = 0$ | $R_\infty = (0,1,1)$ $S_\infty \equiv (0,2,-1)$

Il problema diventa:
coniche tangenti ad r in R_∞, ad s in S_∞



Γ_1: $r \cup s$

Γ_2: r_∞ contata 2 volte

Γ_1: $(X_1 - X_2)(X_1 + 2X_2 - X_0) = 0$

Γ_2: $X_0^2 = 0$

$\mathcal{F}: \lambda(X_1 - X_2)(X_1 + 2X_2 - X_0) + \mu X_0^2 = 0$ $\mathcal{F}: \lambda(x-y)(x+2y-1) + \mu = 0$