$\Sigma : x^3 - 2y^2 - 2z - 4x + 4 = 0 \qquad P = (2, 0, 2)$

$z = \dfrac{x^3 - 2y^2 - 4x + 4}{2} \qquad (u, v) = (2, 0)$

$$\begin{cases} x = u \\ y = v \\ z = \dfrac{u^3 - 2v^2 - 4u + 4}{2} \end{cases} \qquad F(x, y, z) \quad ax + by + cz + d = 0$$

$$\Phi(u, v) = \frac{2cv^2 - 2bv - cu^3 + (4c - 2a)u - 2d - 4c}{2}$$

$$\Phi_u = \frac{-3cu^2 + 4c - 2a}{2}$$

$$\Phi_v = -\frac{4cv - 2b}{2}$$

in $(u, v) = (2, 0)$

$$\begin{cases} \Phi = 0 \\ \Phi_u = 0 \\ \Phi_v = 0 \end{cases}$$

$-\cancel{8c} + \cancel{8c} - 4a - 2d - 4c = 0$

$\dfrac{12c - 4c + 2a}{2} = 0$

$$\begin{cases} 4c + a = 0 \\ b = 0 \end{cases}$$

$$\begin{cases} d = (16k - 4k)/2 = 6k \\ a = -4k \\ c = k \\ b = 0 \end{cases}$$

$\boxed{-4x + z + 6 = 0}$

$\Sigma : x^3 - 2y^2 - 2z - 4x + 4 = 0$

$G :=$

$(2, 0, 2)$

$G_x = 3x^2 - 4 \qquad 8$

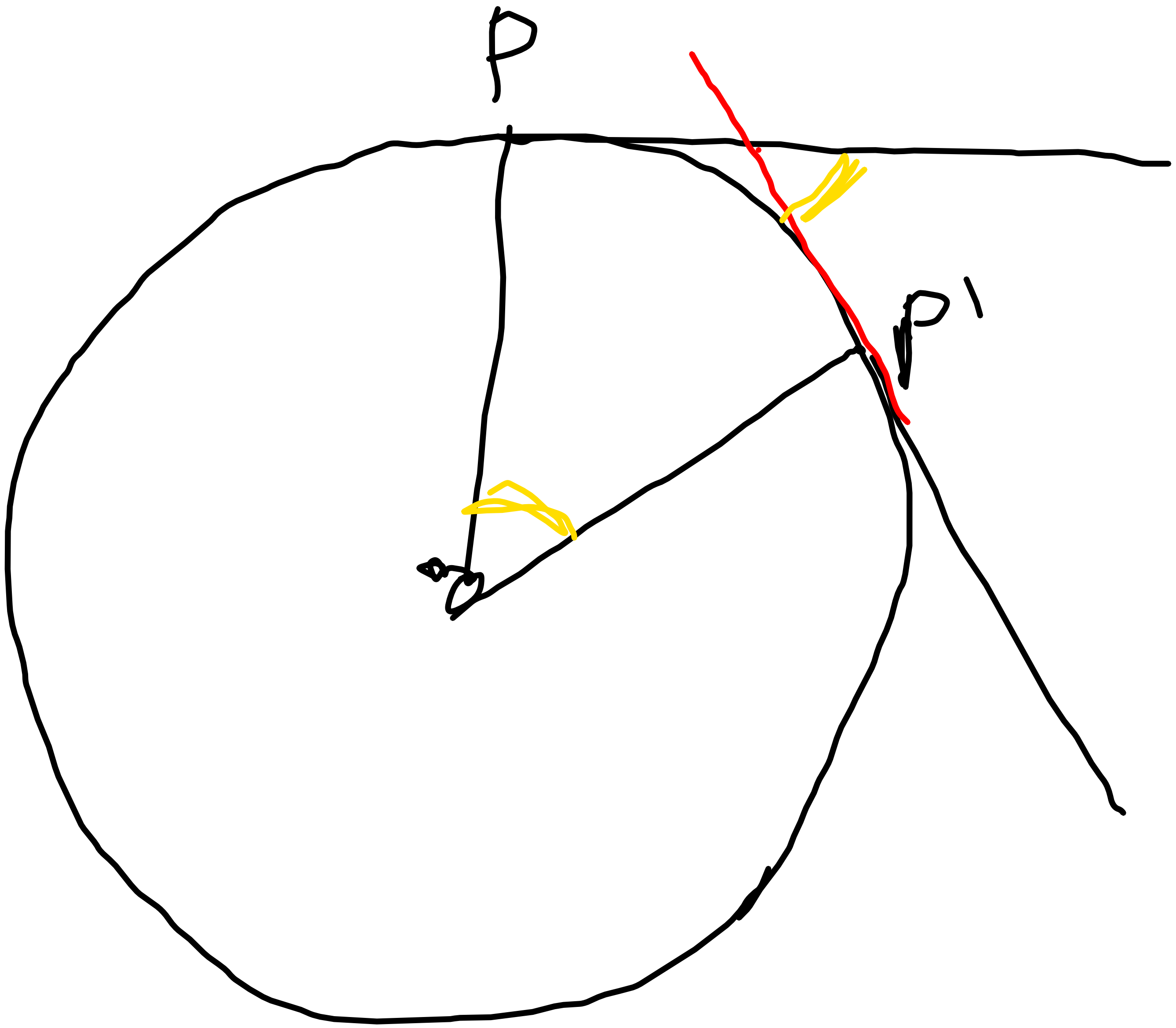
$G_y = -4y \qquad 0$

$G_z = -2 \qquad -2$

$8(x-2) - 2(z-2) = 0$

$8x - 2z - 12 = 0$

$$\boxed{4x - z - 6 = 0}$$


P
P'
θ

$y^2 - 5xy - 2y - x^3 + 2x^2$ $(x+2)^2$

$$\left((x - x_0)\frac{\partial}{\partial x} + (y - y_0)\frac{\partial}{\partial y} \right) f(x_0, y_0)$$

sta per

$$(x - x_0)\frac{\partial f}{\partial x}(x_0, y_0) + (y - y_0)\frac{\partial f}{\partial y}(x_0, y_0)$$

$$\left((x-x_0)\frac{\partial}{\partial x}+(y-y_0)\frac{\partial}{\partial y}\right)^2 f(x_0,y_0)$$

sta per

$$(x-x_0)^2\frac{\partial^2 f}{\partial x^2}(x_0,y_0)+2(x-x_0)(y-y_0)\frac{\partial^2 f}{\partial x\partial y}(x_0,y_0)+(y-y_0)^2\frac{\partial^2 f}{\partial y^2}(x_0,y_0)$$

e così via

Curva $\mathcal{C}: f(x,y)=0$ con $P_0\equiv(x_0,y_0)\in\mathcal{C}$

$$0 = f(x,y) = f(x_0,y_0) = 0$$

$$+ \left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right) f(x_0,y_0)$$

$$+ \left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right)^2 f(x_0,y_0)$$

$$+$$

$$\vdots$$

$$+$$

$$+ \left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right)^{s-1} f(x_0,y_0)$$

$$+ \left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right)^{s} f(x_0,y_0)$$

$$+ \left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right)^{s+1} f(x_0,y_0)$$

$$+$$

Generica retta per P_0:

$$(y-y_0) = k(x-x_0)$$

$$0 = f(x,y) = \cancel{f(x_0,y_0)} = 0$$

$$(y-y_0) = k(x-x_0)$$

$$+ \left((x-x_0)\frac{\partial}{\partial x} + \cancel{(y-y_0)}\,k(x-x_0)\frac{\partial}{\partial y}\right) f(x_0,y_0)$$

$$+ \left((x-x_0)\frac{\partial}{\partial x} + \cancel{(y-y_0)}\,k(x-x_0)\frac{\partial}{\partial y}\right)^2 f(x_0,y_0)$$

$$+ \cdots +$$

$$+ \left((x-x_0)\frac{\partial}{\partial x} + \cancel{(y-y_0)}\,k(x-x_0)\frac{\partial}{\partial y}\right)^{s-1} f(x_0,y_0)$$

$$+ \left((x-x_0)\frac{\partial}{\partial x} + \cancel{(y-y_0)}\,k(x-x_0)\frac{\partial}{\partial y}\right)^{s} f(x_0,y_0)$$

$$+ \left((x-x_0)\frac{\partial}{\partial x} + \cancel{(y-y_0)}\,k(x-x_0)\frac{\partial}{\partial y}\right)^{s+1} f(x_0,y_0)$$

$$+ \cdots$$

$$= \boxed{(x-x_0)\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right) f(x_0,y_0) + (x-x_0)^2\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 f(x_0,y_0) + \cdots + (x-x_0)^{s-1}\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^{s-1} f(x_0,y_0)}$$

$$\boxed{+ (x-x_0)^{s}\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^{s} f(x_0,y_0)}$$

$$+ (x-x_0)^{s+1}\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^{s+1} f(x_0,y_0) + \cdots$$

P_0 è s-plo se la generica retta per P_0 ha raccolte in P_0 s intersez.

una retta particolare ha raccolte almeno s+1 intersez.

$\mathcal{C}: x^3 + y^3 - 3xy = 0$ $\quad x = 0 \cup y = 0$

$F(x,y) =$

		$(0,0)$
$F_x = 3x^2 - 3y$		0
$F_y = 3y^2 - 3x$		0

$\Rightarrow$ O è multiplo

$y = kx$

$ax = by$

$F_{xx} = 6x$	0
$F_{xy} = -3$	-3
$F_{yy} = 6y$	0

$\Rightarrow -3 \neq 0$ O è doppio

tangenti nel punto doppio

$$\frac{\partial^2 f}{\partial x^2}(0,0) + 2k\frac{\partial^2 f}{\partial x \partial y}(0,0) + k^2\frac{\partial^2 f}{\partial y^2} = 0$$

$$b^2\frac{\partial^2 f}{\partial x^2}(0,0) + 2ab\frac{\partial^2 f}{\partial x \partial y}(0,0) + a^2\frac{\partial^2 f}{\partial y^2} = 0$$

$-6k = 0$

$-6ab = 0$ $\quad a = 0 \quad b = 0$

$\mathcal{C}: y = x^2$ $\mathcal{C}': y = -x^3$ $\mathcal{L}$ luogo delle int. delle tang. a $\mathcal{C}$ e $\mathcal{C}'$ in punti di uguale ascissa

$\mathcal{C}: \begin{cases} x = u \\ y = u^2 \end{cases}$ $P_u \equiv (u, u^2)$ $t_u: y - u^2 = 2u(x-u)$

$\mathcal{C}': \begin{cases} x = u \\ y = -u^3 \end{cases}$ $Q_u \equiv (u, -u^3)$ $t'_u: y + u^3 = -3u^2(x-u)$

$\mathcal{L}: \begin{cases} u^2 - 2xu + y = 0 \\ 2u^3 - 3xu^2 - y = 0 \end{cases}$ $y(4y^2 - 3x^2y + 6xy + y - 4x^3) = 0$

den: $4(y-x^2)^2$

$y(4y^2 - 3x^2y + 6xy + y - 4x^3) = 0$

$F(x,y) =$

$$\begin{cases} F = 0 \\ F_x = 0 \\ F_y = 0 \end{cases}$$

$F_{xx} = -6y - 24x$ $(-1,1)$ 18

$F_{xy} = -6x + 6$ 12

$F_{yy} = 8$ 8

$F_x = -6xy + 6y - 12x^2 = 0$

$F_y = 8y - 3x^2 + 6x + 1 = 0$ $y = \frac{3x^2 - 6x - 1}{8}$

$$\begin{cases} F_x = 0 \\ F_y = 0 \end{cases} \quad - \frac{3(x+1)^2(3x+1)}{4}$$

$x = -1 \Rightarrow y = 1$

$x = -\frac{1}{3} \Rightarrow y = \frac{1}{6}$

$(-1, 1)$ multiplo

$\cancel{\left(-\frac{1}{3}, \frac{1}{6}\right)}$

$18 + 24k + 8k^2 = 0$ $4k^2 + 12k + 9 = 0$

$k = \frac{-6 \pm \sqrt{36-36}}{4} = -\frac{3}{2}$ unica tang (doppia)

$\boxed{y - 1 = -\frac{3}{2}(x+1)}$