A : adjacency matrix of graph G.

Claim: in $A^k = (b^i_j)$ b^i_j is the # of k-walks from v_i to v_j

Proof (by induction)

Inductive premise (k=1)

A 1-walk is a sequence uev, where e is an edge between u and v. Then A is actually the matrix giving a^i_j as the # of 1-walks from v_i to v_j

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Inductive step (for simple graphs, but easily adaptable for any graph)

Inductive hypothesis: $A^{h-1} = (c^i_j)$ has c^i_j as the # of $(h-1)$-walks from v_i to v_j

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Inductive thesis: $A^h = (d^i_j)$ has d^i_j as the # of h-walks from v_i to v_j

Every h-walk from v_i to v_j is so built:

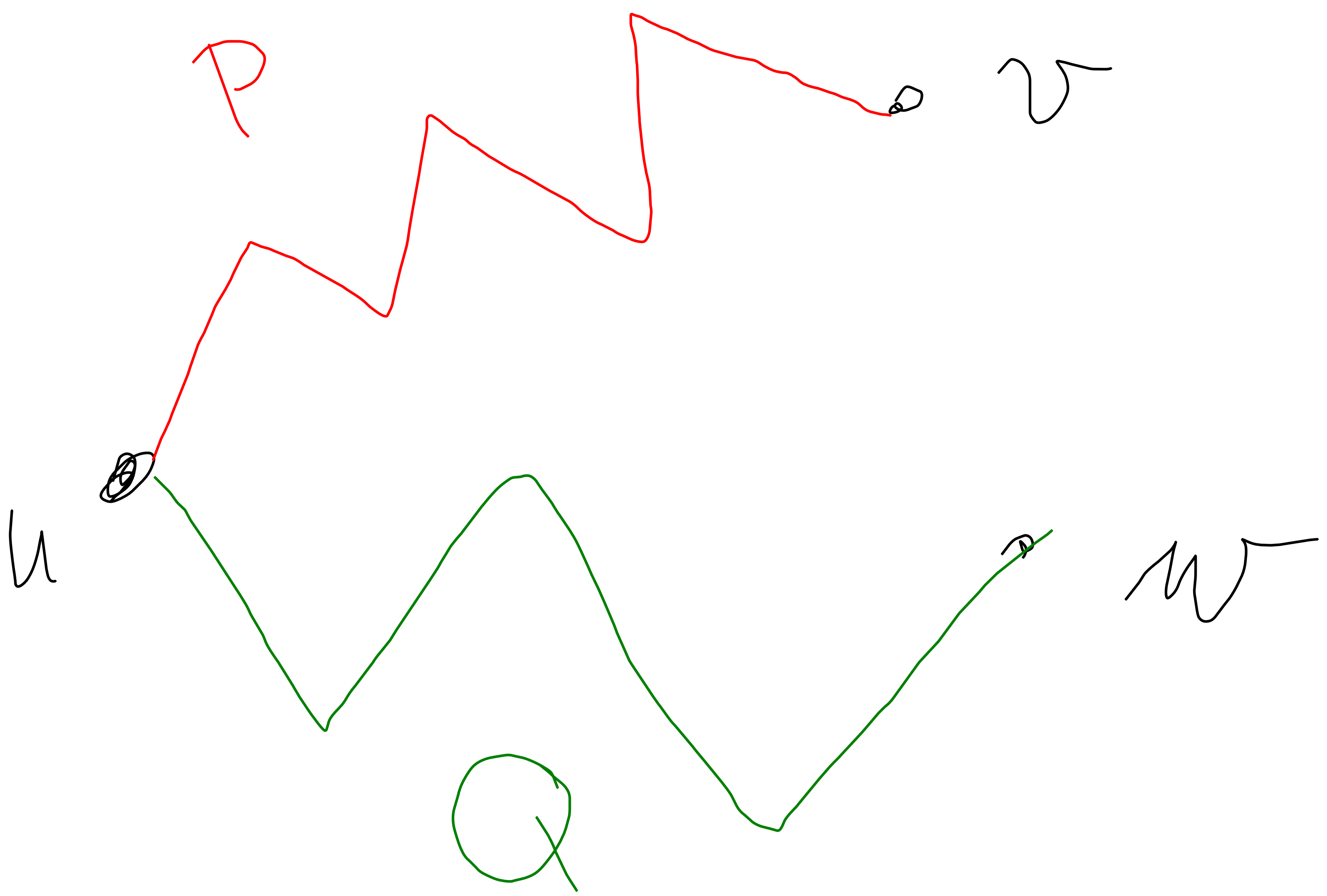
an $(h-1)$-walk w_s from v_i to v_s, followed by the edge $v_s v_j$ IF THAT EDGE EXISTS.

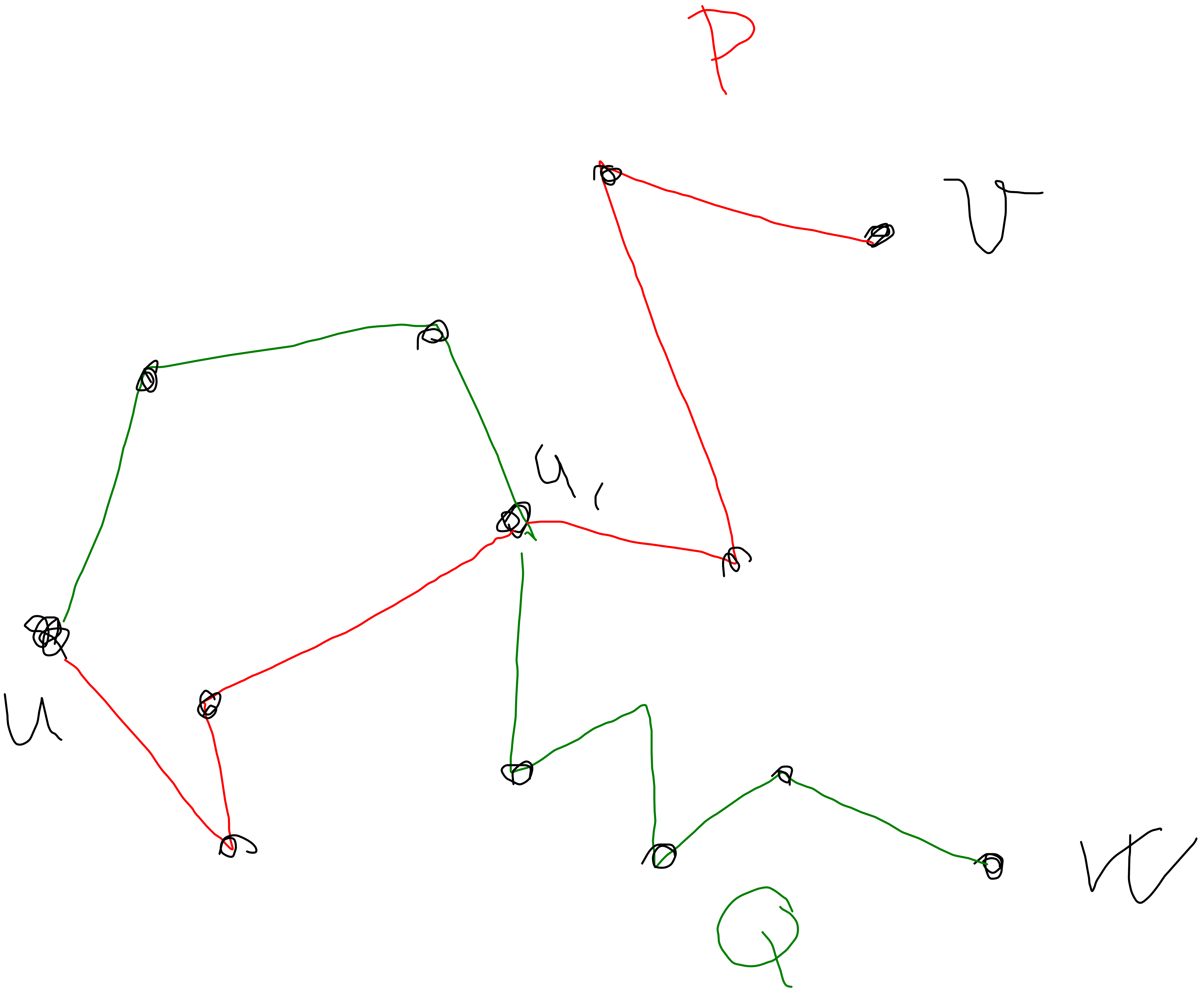
So the number of h-walks from v_i to v_j is equal to:

of $(h-1)$ walks from v_i to v_1 (if an edge between v_i and v_1 exists, i.e. if $a^i_1 = 1$) + # of $(h-1)$ walks from v_i to v_2 (if $a^i_2 = 1$) + ... + # of $(h-1)$ walks from v_i to v_ν (if $a^i_\nu = 1$) =

(by inductive hypotesis)

$c_1^i \cdot a_j^1 + c_2^i \cdot a_j^2 + \cdots + c_\nu^i a_j^\nu = d_j^i$ of A^h

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