

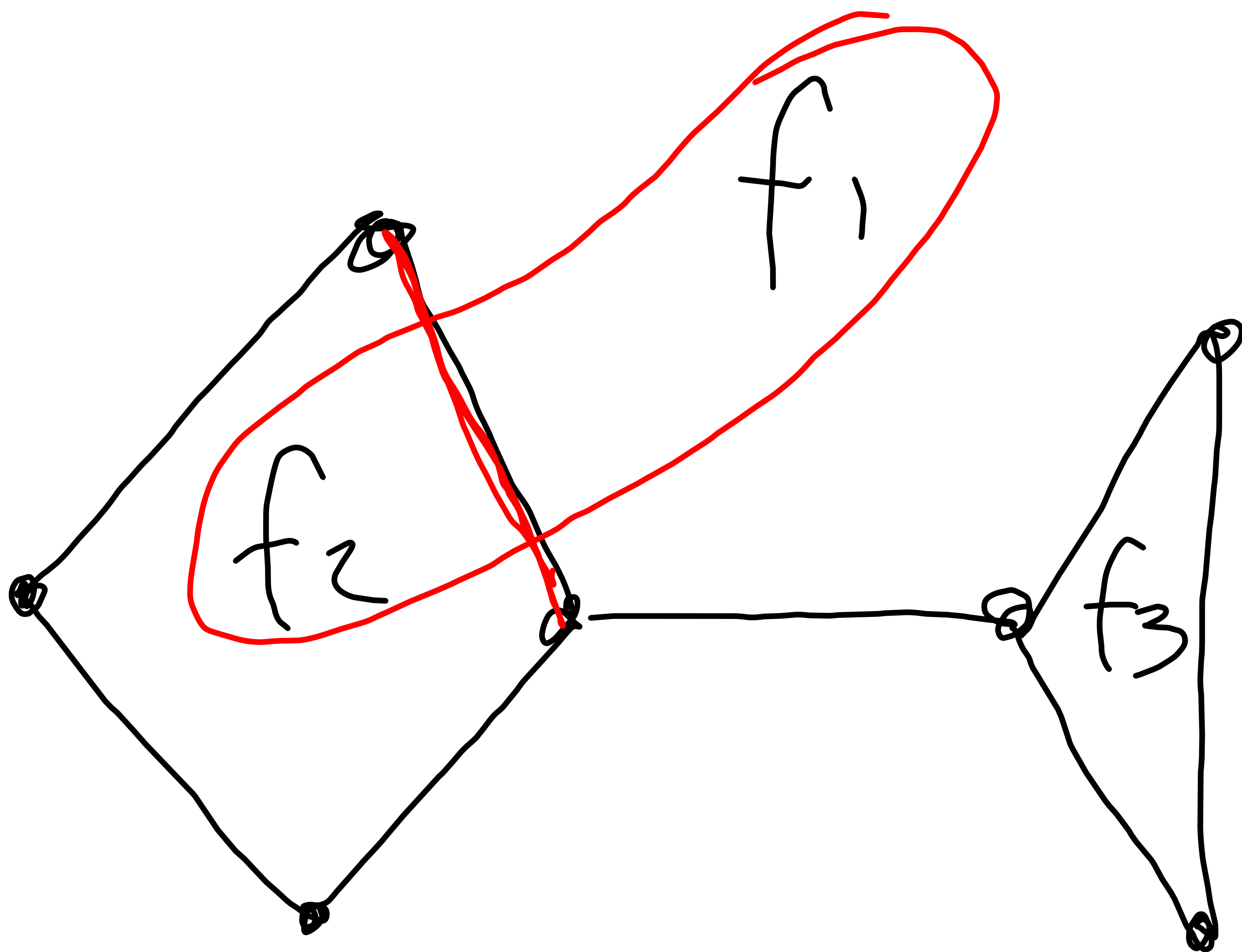


Inductive premise

$\phi(G) = 1 \implies G$  is a tree

$$v(G) - 1 = \varepsilon(G)$$

$$\begin{aligned} v(G) - \varepsilon(G) + \phi(G) &= v(G) - (v(G) - 1) + 1 = \\ &= \cancel{v(G)} - \cancel{v(G)} + 1 + 1 = 2 \end{aligned}$$



$$\begin{aligned}
 \nu(G-e) - \varepsilon(G-e) + \phi(G-e) &= 2 \\
 \nu(G) - (\varepsilon(G) - 1) + (\phi(G) - 1) &= 2 \\
 \nu(G) - \varepsilon(G) + 1 + \phi(G) - 1 &= 2 \\
 \nu(G) - \varepsilon(G) + \phi(G) &= 2
 \end{aligned}$$

$$+ d(f_1) \geq 3$$
$$+ d(f_2) \geq 3 +$$

⋮

$$+ d(f_\phi) \geq 3 +$$

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$$\sum_{i=1}^{\phi} d(f_i) \geq 3\phi$$



$$\frac{2\varepsilon}{3} \geq \phi$$

$$v - \varepsilon + \frac{2\varepsilon}{3} \geq v - \varepsilon + \phi = 2$$

$$v - \frac{v}{3} \geq 2$$

$$\frac{2v}{3} \geq v - 2$$

$$v \geq 3v - 6$$

$$\sqrt{x} \leq 6x - 12$$

$$\sqrt{x} \leq 6 - \frac{12}{x}$$