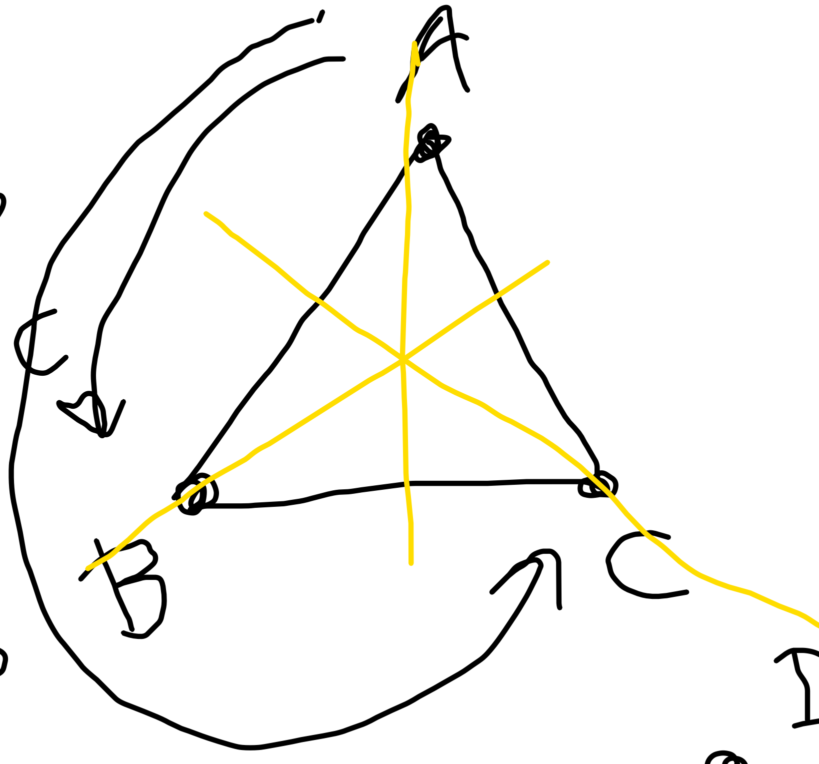
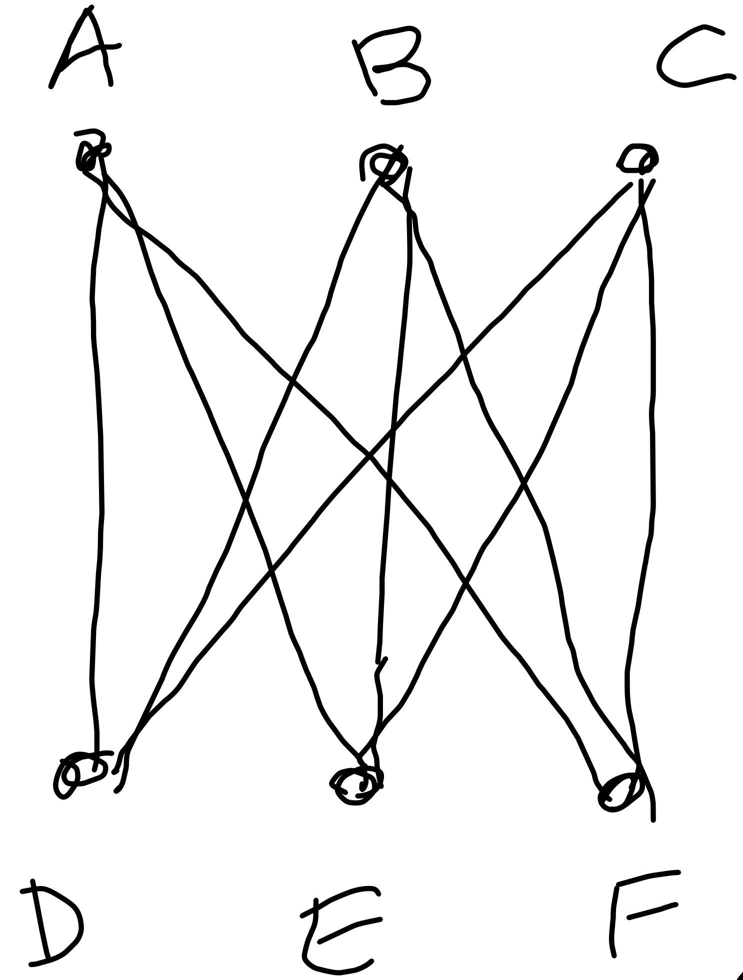
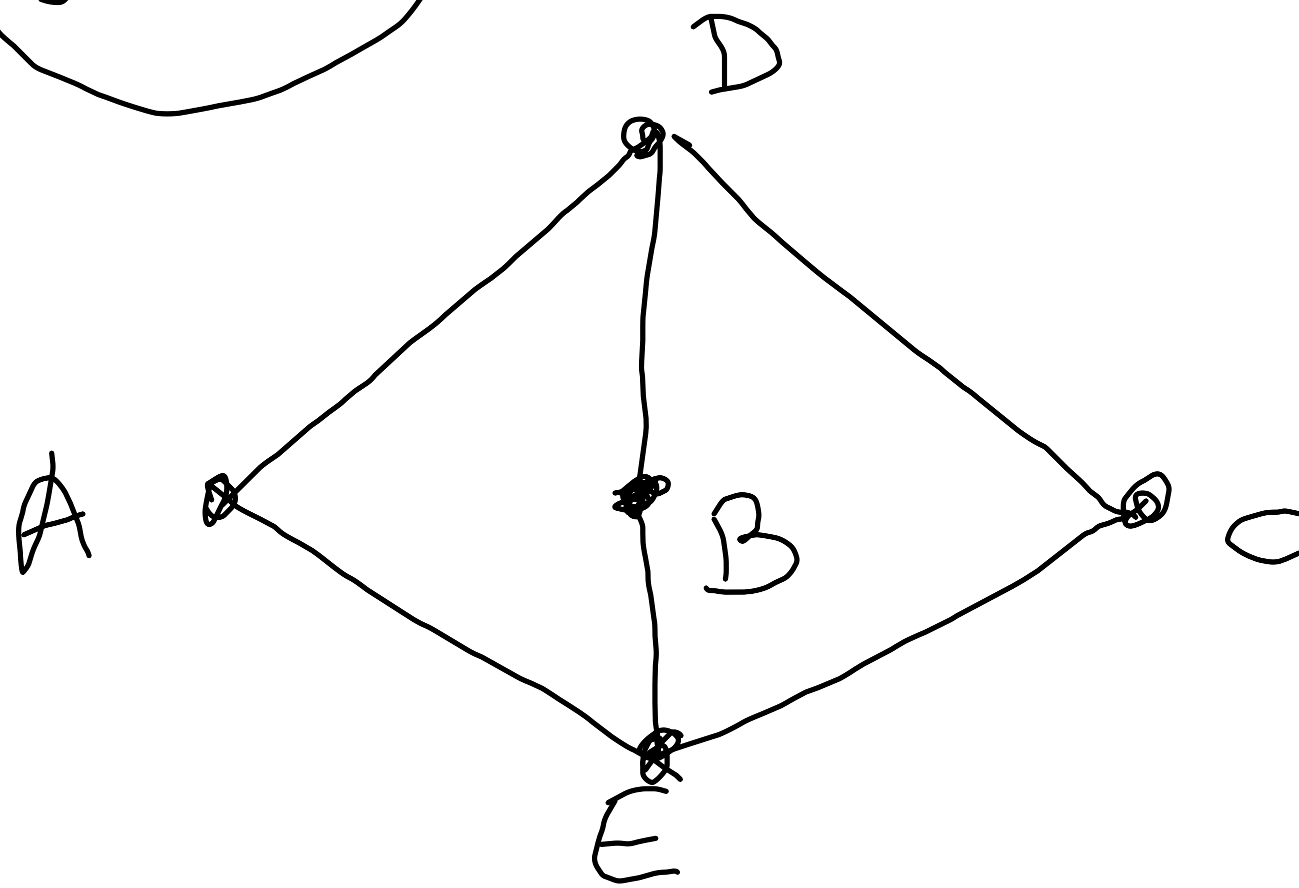
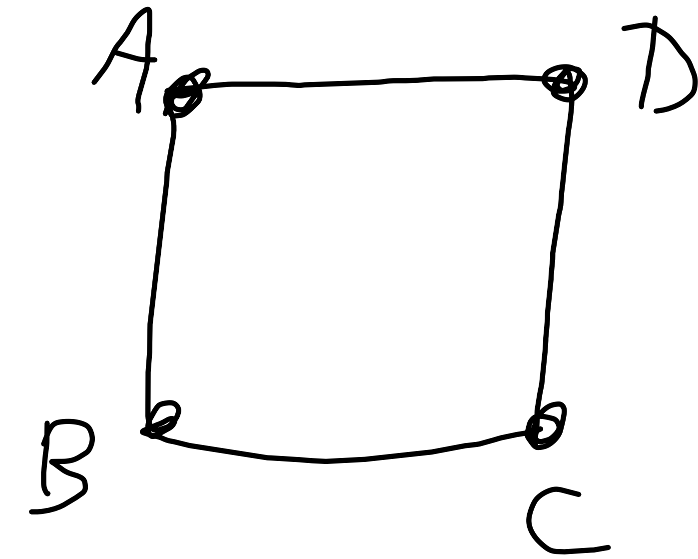
$A \mapsto A$
$B \mapsto B$
$C \mapsto C$



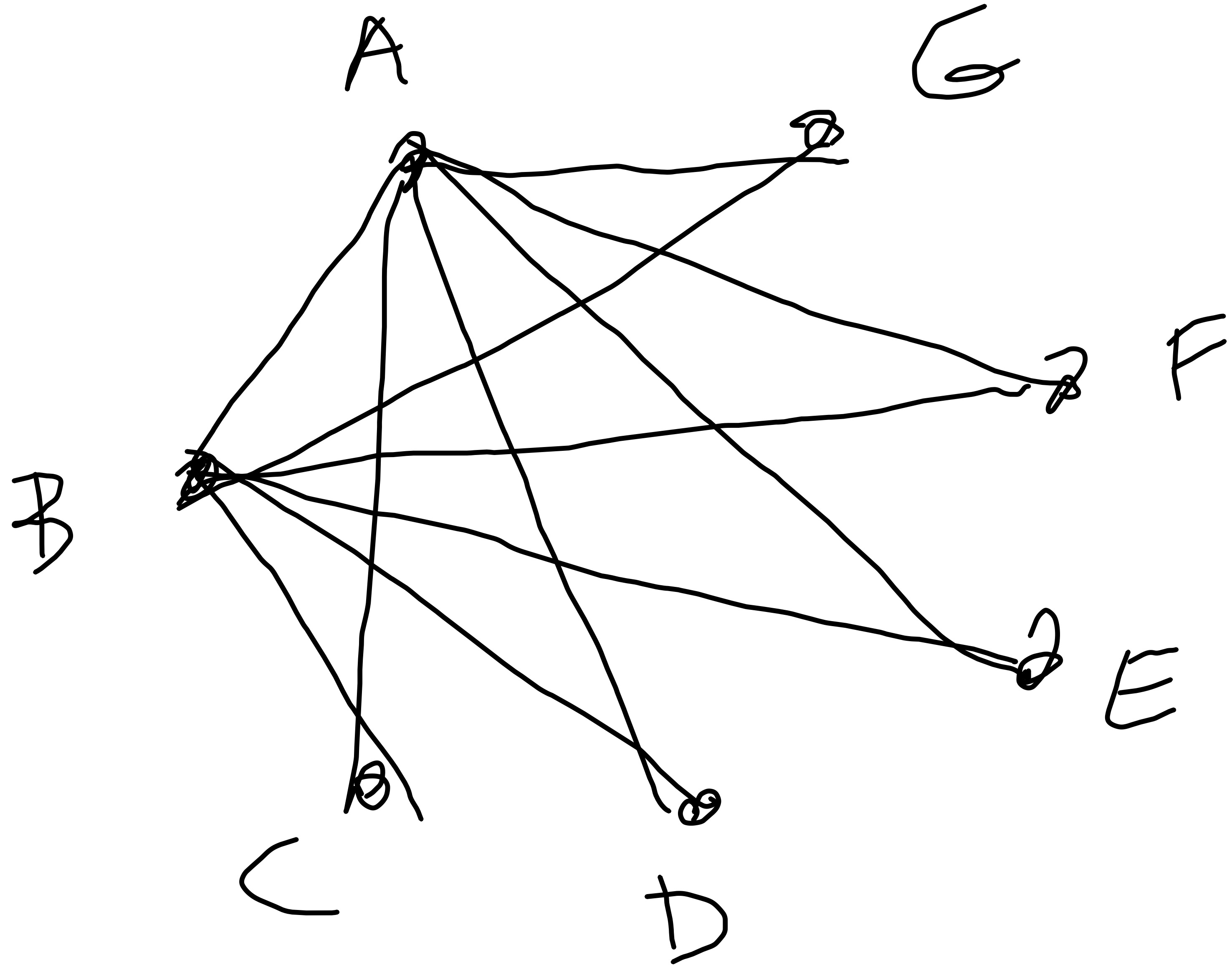
$A \mapsto B$
$B \mapsto C$
$C \mapsto A$







$(6, 6, 5, 4, 3, 3, 1)$



Sum of numbers is even

A

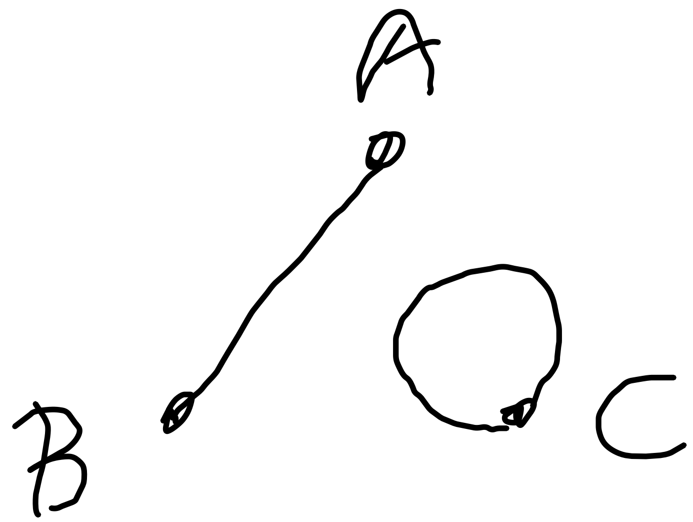
$\exists$ a graph with those degrees

B

Giulia proves: $(\neg A) \Rightarrow (\neg B)$

this is logically equivalent to: $B \Rightarrow A$

(1, 1, 1, 2)



(8, 5, 6, 1, 4, 7, 3)

4 loops

2 loops

3 loops

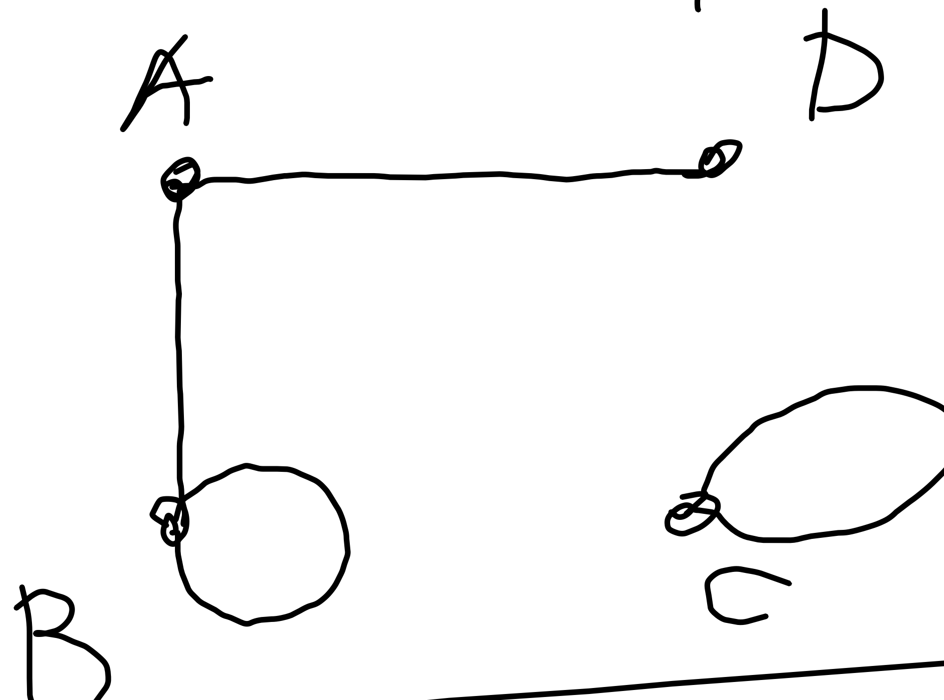
no loops

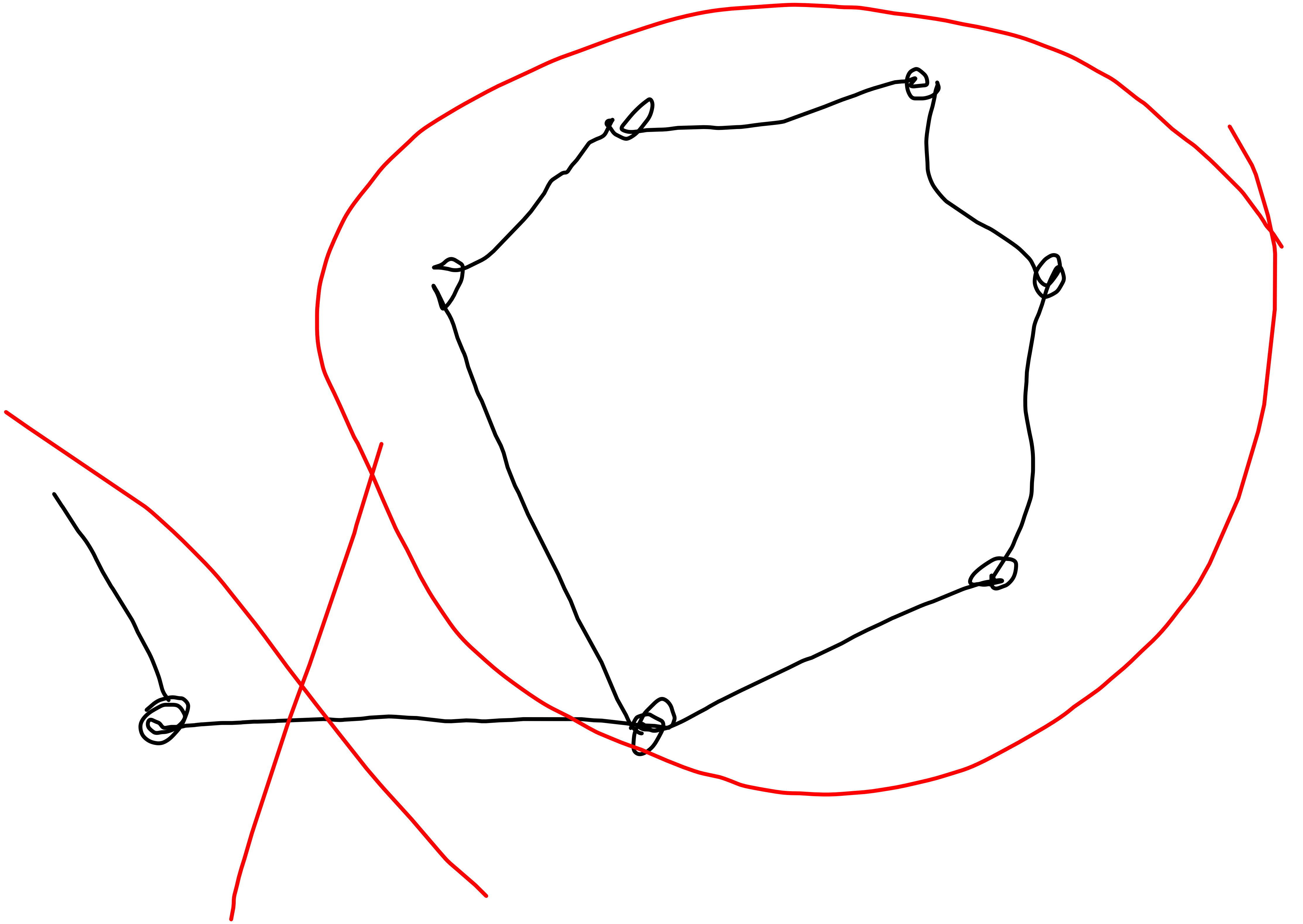
2 loops

3 loops

(2, 3, 2, 1)

A B C D





$A \Rightarrow B$ this is defined as equivalent

A implies B to:

$$(\neg A) \vee B$$

A	B	$\neg A$	$A \Rightarrow B$ $(\neg A) \vee B$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

PROP – Whatever the set X,

$\emptyset \subseteq X$

What is the meaning of $\subseteq$?

DEF: $A \subseteq B$ is defined by the implication

$(x \in A) \Rightarrow (x \in B)$

PROOF of : $(x \in \emptyset) \Rightarrow (x \in X)$?

YES: $(x \in \emptyset)$ is False

PROP.

Given an implication $A \Rightarrow B$, the counterpositive of it, i.e. $(\neg B) \Rightarrow (\neg A)$ is equivalent.

PROOF

$A \Rightarrow B$ equiv. to $(\neg A) \vee B$

Analogously,

$(\neg B) \Rightarrow (\neg A)$ equiv. to $\neg(\neg B) \vee (\neg A)$ equiv. to

to $B \vee (\neg A)$ equiv. to $(\neg A) \vee B$ equiv. to $A \Rightarrow B$

Proof by induction

We want to prove a certain property P for a graph with n vertices (or n edges, or n cycles, ..)

The proof is made of 2 parts: Inductive premise and Inductive step.

<u>Inductive premise</u>: We prove property P for a very small n (0 or 1 or 2)

<u>Inductive step</u>: We assume the Inductive hypothesis: "P holds for $k-1$", and we prove the Inductive thesis: "P holds for k".

An example: prove that the complete graph K_n with n vertices has $n(n-1)/2$ edges.

Inductive premise. $n=2$ K_2:  2 vertices, 1 edge

$$1 = \frac{2\cdot(2-1)}{2}$$

Inductive step. Call $e(n)$ the number of edges of K_n (it is what I want to prove equal to $n\cdot(n-1)/2$)

Build the complete graph on $k-1$ vertices K_{k-1}; it has $e(k-1)$ edges. Now add a further vertex and join it to all vertices of K_{k-1}: you get K_k: then $e(k)=e(k-1)+k-1$

$e(k) = e(k-1) + k-1$

\# of edges of K_k

\# of edges of K_{k-1}

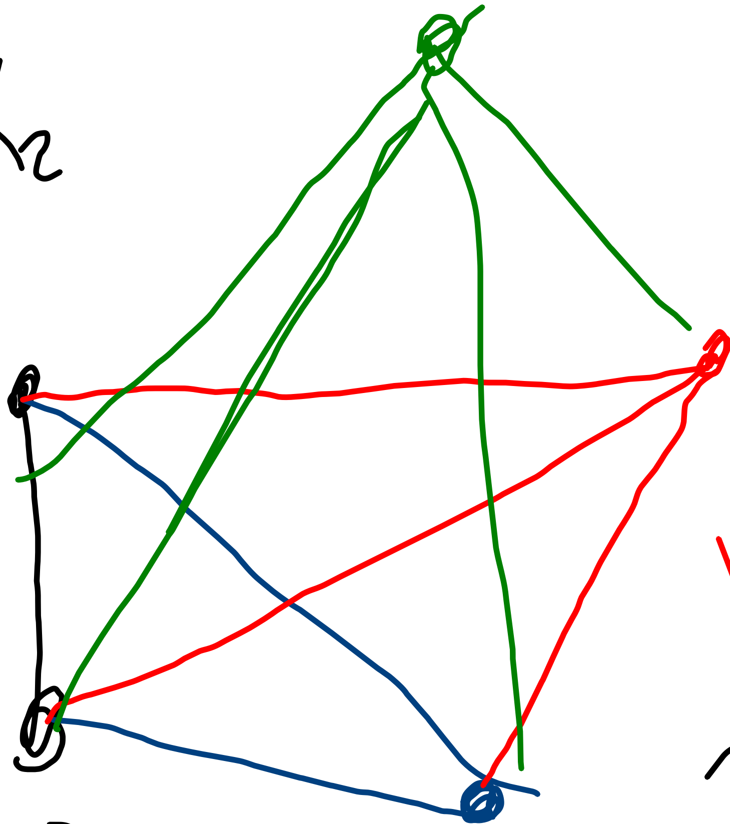
\# of edges we added

Now I apply the Inductive hypothesis: $e(k-1) = (k-1)(k-2)/2$

$$e(k) = e(k-1) + k-1 = \frac{(k-1)(k-2)}{2} + k-1 = \frac{(k-1)(k-2)+2(k-1)}{2} =$$

$$= \frac{(k-1)(k-\cancel{2}+\cancel{2})}{2} = \frac{k(k-1)}{2}$$

K_2



$$\frac{2\cdot(2-1)}{2}$$

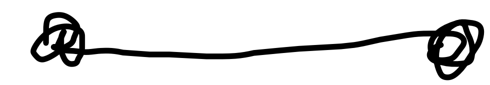
K_3

$$\frac{2\cdot(2-1)}{2}+2=3=\frac{3\cdot(3-1)}{2}$$

K_4

$$\frac{3\cdot(3-1)}{2}+3=6=\frac{4(4-1)}{2}$$

K_2



$\nu = 2$

$\varepsilon = 1 = 2 - 1$

G tree with ν vertices

$e = uv$ in $E(G)$

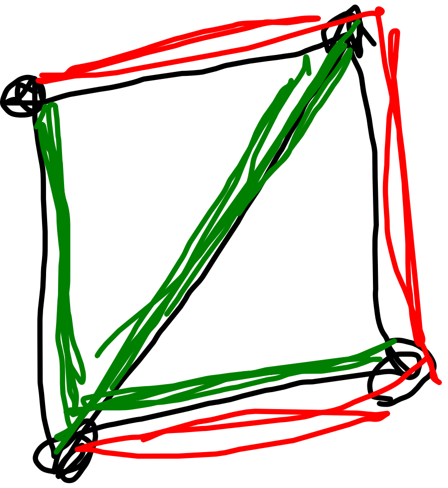
$G - e$ is made of two components: G_1 and G_2

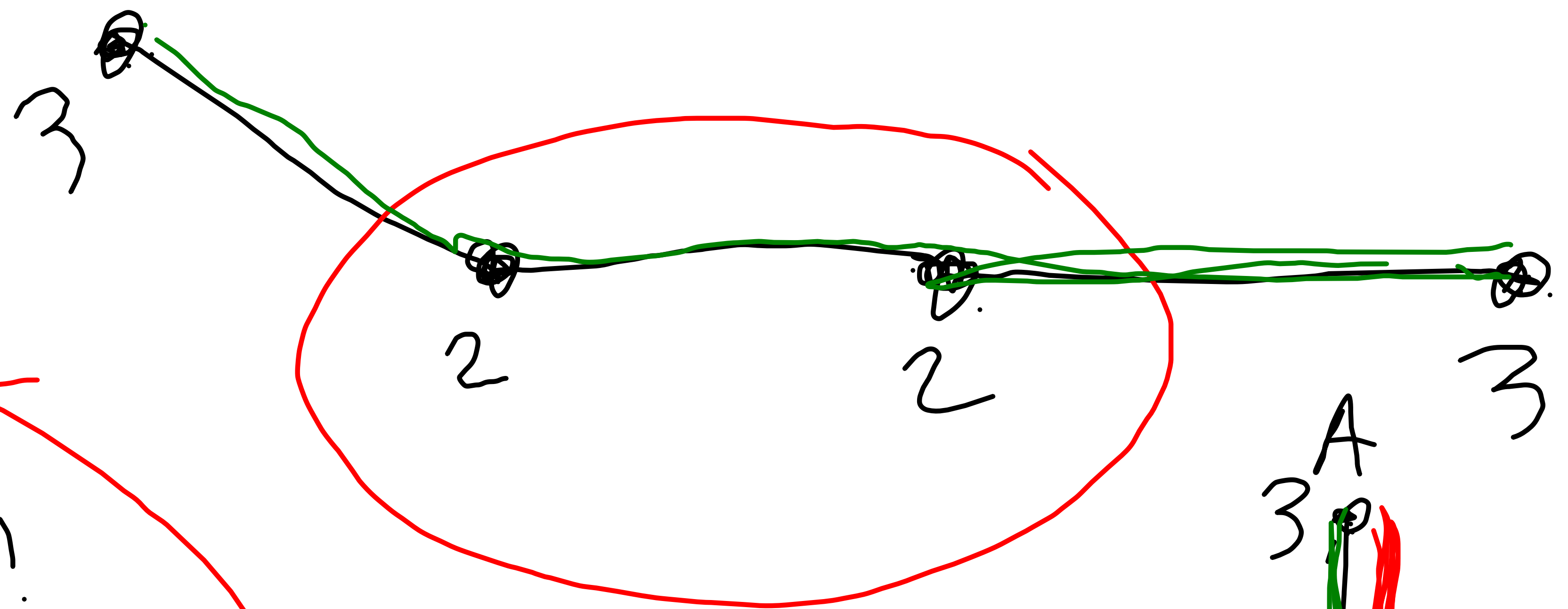
Both G_1 and G_2 are connected and acyclic, so they are trees. Both have less than ν vertices so we can apply the inductive hypothesis:

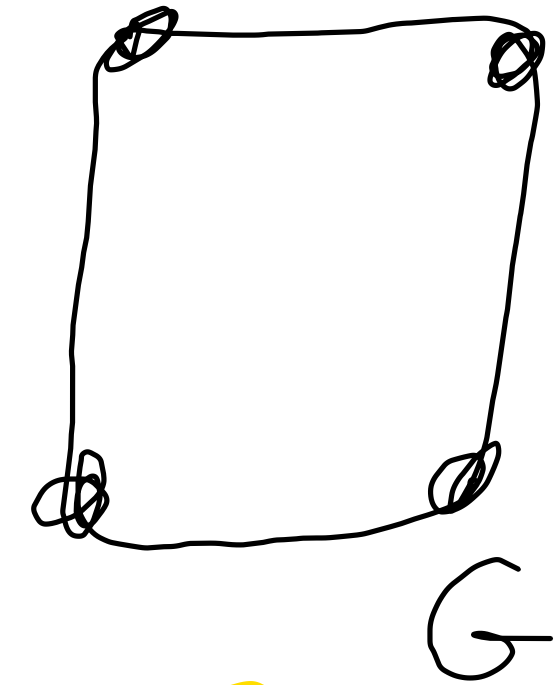
$\varepsilon(G_1) = \nu(G_1) - 1 \qquad \varepsilon(G_2) = \nu(G_2) - 1$

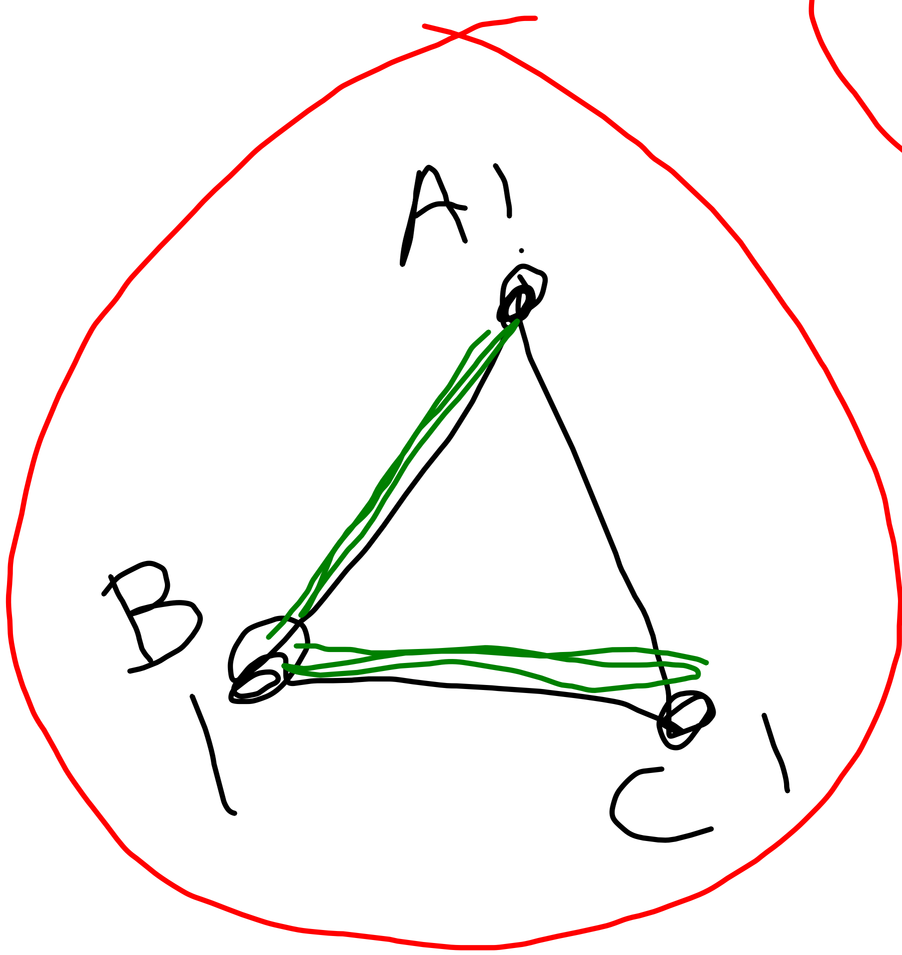
$$\varepsilon(G) = \varepsilon(G_1) + \varepsilon(G_2) + \underset{e}{1} = \nu(G_1) \cancel{-1} + \nu(G_2) - 1 \cancel{+1} =$$

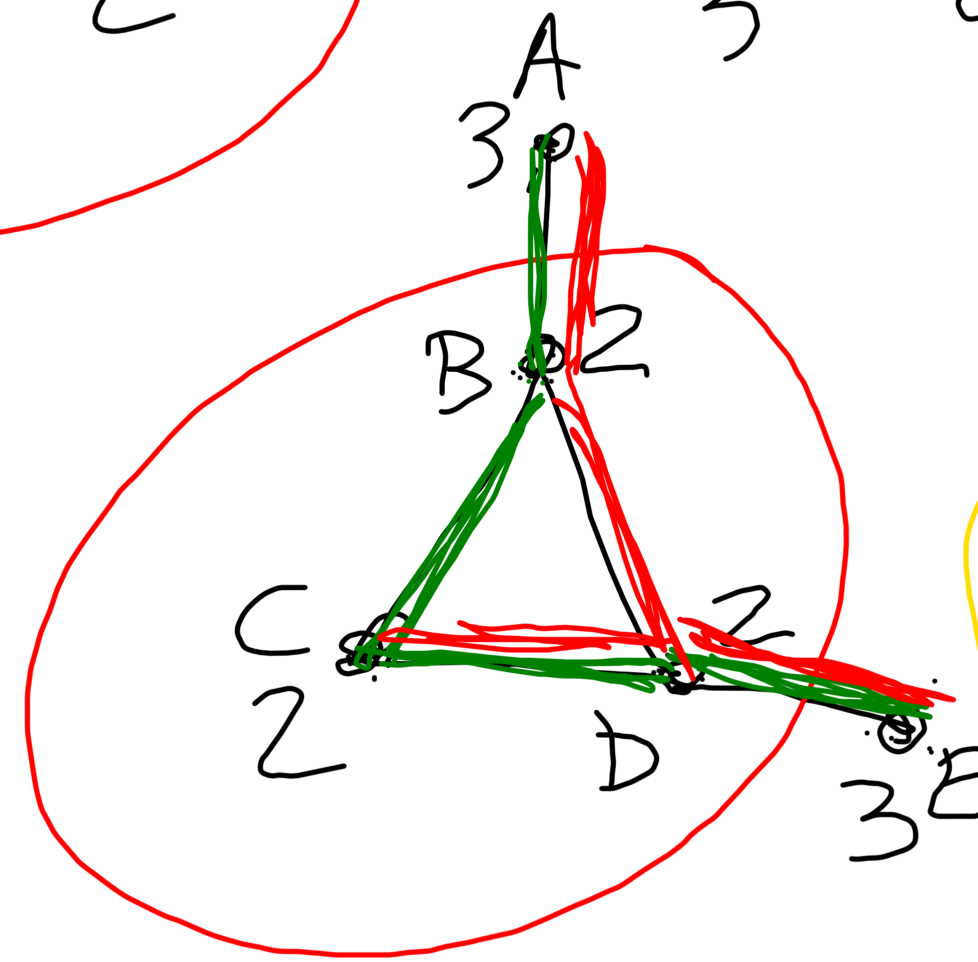
$$= \nu(G) - 1$$

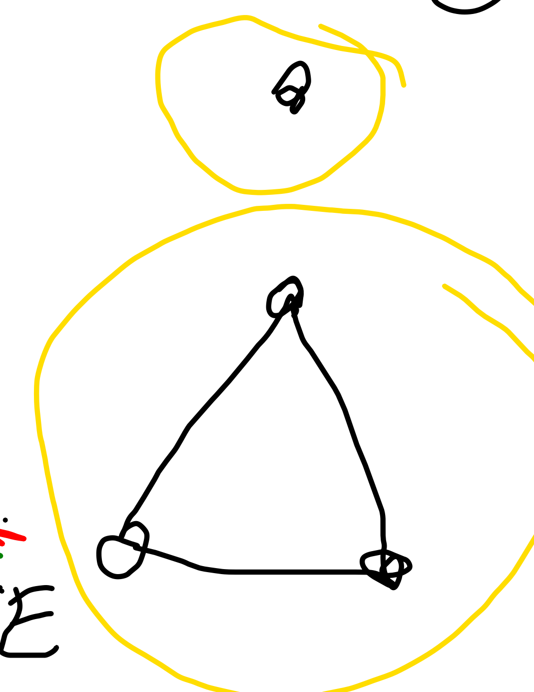


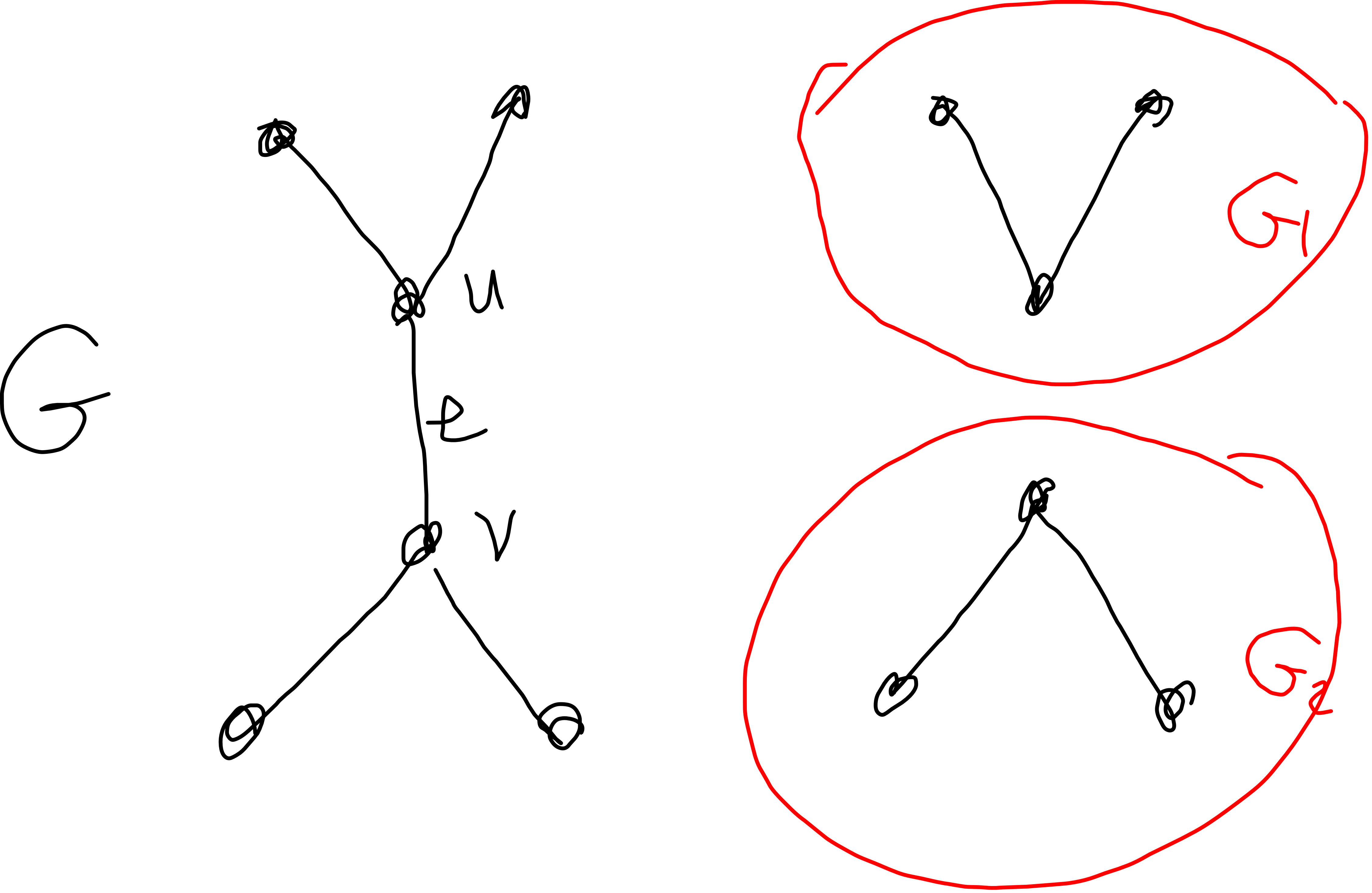

3
2
2
3


G


A 1
B
1
C 1


A
3
B 2
C
2
2
D
3 E




G
u
e
v
G1
G2