Exercise sheet No. 12

Deadline: 28/01/2020.
You should hand only exercises 0.1, 0.2 and 0.3.

Exercise 0.1 (5 points)**.** Compute the integral simplicial volume of n-spheres and of oriented closed connected surfaces.

Exercise 0.2 (5 points)**.** Let M and N be two oriented closed connected manifold of dimension m, n respectively. Prove that

$$\|M\| \cdot \|N\| \leq \|M \times N\| \leq \binom{n+m}{m} \|M\| \cdot \|N\| \ .$$

Exercise 0.3 (5 points)**.** Prove that

(i) The function $\|\cdot\|_{\mathbb{Z}} \colon H_n(M, \partial M; \mathbb{Z}) \to \mathbb{R}$ is not a semi-norm.

Discuss

(ii) The relation between the integral simplicial volume of the base space N and the covering space M for a degree d covering map $f \colon M \to N$.

Assume to know that $\|\Sigma_g\| \geq 2|\chi(\Sigma_g)|$.

(iii) Prove that $\|\Sigma_g\| = 2|\chi(\Sigma_g)|$.

Exercise 0.4. Let $p \colon E \to B$ be a locally trivial fiber bundle with fiber F, where F, E, B are orientable compact connected manifolds. Assume $\dim(F) \geq 1$ and that $i_*(\pi_1(F)) \leq \pi_1(E)$ of the fundamental group of F under the inclusion $i \colon F \hookrightarrow E$ is amenable. Prove that

(i) $\|E\| = 0$.

Then,

(ii) Construct explicitly an example of such manifold E.