Project 2) Modeling a Bungee Jump using ODE

Problem Description

Bungee jumping is a famous extreme sport that involves jumping from a high and fixed structure like bridge or building while attached to a large elastic cord. The bungee jumping is a typical example of dynamical system where physicists are interested to study andanalyze its phenomenon. In brief, this experience can be described as: the bungee jumper jumps off a bridge and then falls down until the elastic bungee cord slows his descent to a stop, before pulling him back up. The jumper then oscillates up and down until all the energy is dissipated. The study of this sport in terms of safety and risk require using physics laws and theory of differential equations.
The rope used is different from case to case and depends on both the type of jump and the jumper: in particular, the elastic constant of the rope must be chosen based on the weight of the subject, the length of the rope and the height of the launching platform. with respect to the ground. In this exercise, the goal will be to determine which of the three bungee cords of the three available is the most suitable for a jumper with a mass of
m = 90Kg who jumps from a bridge that is 80m high.

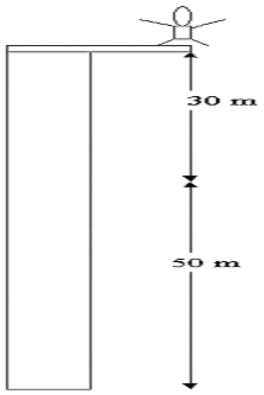
The elastic constants of the three cases are:

cord	**elastic constant (K)**
A	= 5 N/m
B	= 40 N/m
C	= 500 N/m

Air resistence:

$R = a1*v - a2 *|v|* v$ where $a1=1$ and $a2=1$

Knowing that the launch is carried out from a height of 80m and that the length of the three cables is 30m, we deduce that for a first stretch equal to 30m the jumper will be subject to the weight force and air resistance only. Only after 30m the rope, considered purely elastic, will develop a return force proportional to its elongation according to the elasticity constants $K_{a,b,c}$.



Description of the mathematical model

In order to solve this problem, we need to:

1. Determine the forces acting on the body.
2. Draw the free body diagram.
3. Apply Newton's second law.
4. Solve the ODE numerically.

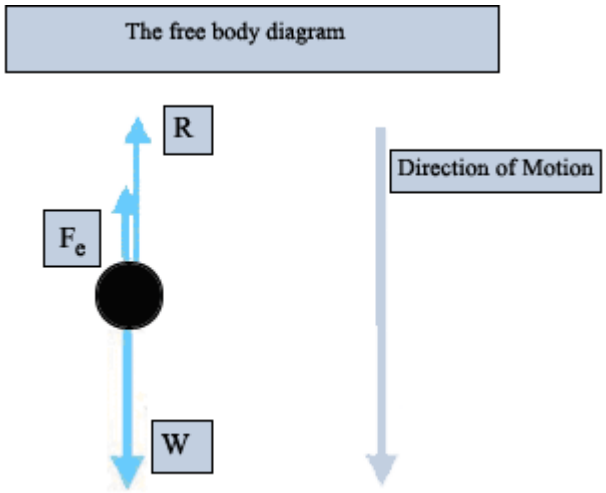
1. Determine the forces acting on the body.

Weight (W): directed downwards
W= m*g
m = 90 Kg
g = 9.8 m/s^2

Air Resistance (R): force that opposes motion
R = a1*v+a2 *|v|* v with a1=1, a2=1
v=dx/dt

Force of the elastic cord (Fe):
active return force if x greater than the length of the rope
Fe = k*(x-30) if x>0
0 if x<0

2. Draw the free body diagram.



3. Apply Newton's second law F = m * a

W – R -Fe = m*a
mg – Fe - a1*v+a2 *|v|* v=m*a
Let x be the body position.

- For x < 30, we get the ODE m*a = W-R

$$\frac{d^2x}{dt^2} = g - \frac{a_1}{m}\frac{dx}{dt} - \frac{a_2}{m}\left|\frac{dx}{dt}\right|\frac{dx}{dt}$$
$$x(0) = 0, \frac{dx(0)}{dt} = 0$$

- For x > 30, we get the ODE m*a = W-R-Fe

$$\frac{d^2x}{dt^2} = g - \frac{Fe}{m} - \frac{a_1}{m}\frac{dx}{dt} - \frac{a_2}{m}\left|\frac{dx}{dt}\right|\frac{dx}{dt}$$
$$x(t_{30}) = 30, \frac{dx(t_{30})}{dt} = v_{30}$$

4. Numerically solve the two interval times of the launch and plot the three complete trajectories (free fall + elastic retraction) as the cable varies. Use and compare the accuracy of ode45 solvers, Heun snd explicit, implicit euler methods.

History of Bungee Jumping

Bungee jumping activity came from a small Island in Australia [1]. A tribe called “Bunlap” practice this sport where the jump from towers. They used vine instead of cord. The Bunlapmen practice this sport according to legend in this tribe to discover the trickery of their wife! In 1970, a reporter from National Geographic Magazine named Kal Muller went to this island and asked this tribe to teach him this activity. Later in 1979, four members of oxford danger sports read Muller’s stories. After that, they went to Clifton Bridge in Bristol, UK and jump using bungee cords instead of vines. Today bungee jumping is practicing worldwide.

[1] J.J. Buckley and L.J. Jowers. Simulating Continuous Fuzzy Systems. Studies in Fuzziness and Soft Computing (Vol.177). Springer-Verlag, Heidelberg, Germany, 2005