Project 10:

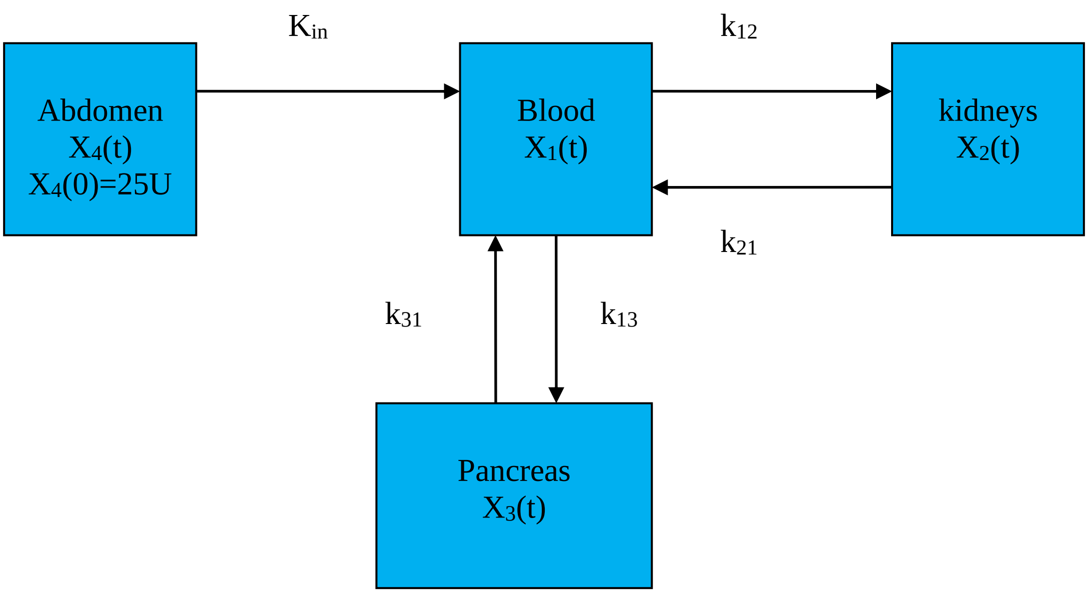
"COMPARTMENT MODEL FOR THE STUDY OF THE KINETICS OF INSULIN CIRCULATION IN THE BLOOD"

Problem Definition

A significant amount of diabetic patients use daily insulin injections to keep their blood sugar stable. Increasing the amount of insulin in the blood decreases the amount of sugar in the blood. Several methods exist for monitoring blood sugar, however, the inability to achieve a predictable blood sugar over time is a frustrating problem that constantly plagues diabetics.

Problem Analysis

The model shown in the Figure below represents the flux of insulin through specific parts of the body. For analytical purposes, this model does not take into account other factors, such as glucose, that may normally affect the concentration of insulin throughout the body. Using this model, it is your goal to predict the level of insulin in the blood over time after an insulin injection.
The Figure shows the flow of insulin in the body. A patient is first given an insulin injection, usually in the abdomen or upper arm. The insulin then dissolves into the bloodstream. Once in the blood, insulin flows to and from the kidneys, as well as to and from the pancreas.



We denote by $x_i(t)$ the amount of insulin in compartment i at time t. We will denote by k_{ij} the proportionality factor from j to i and by k_{ji} that from i to j.

Let:
• x1(t) be the concentration of insulin in the blood.
• x2(t) be the concentration of insulin in the kidneys.
• x3(t) be the concentration of insulin in the pancreas.
• Let x4(t) be the concentration of insulin in the abdomen. Initialize x4(0) = 25 Units (U) to be the amount of insulin initially present in the abdomen due to the injection.
• k_{in} = 2 U/hr be the flow rate of insulin from the abdomen into the blood.
• k_{12} = 1 U/hr be the flow rate of insulin from the blood into the kidneys.
• k_{21} = 1.5 U/hr be the flow rate of insulin from the kidneys into the blood.
• k_{13} = 2 U/hr be the flow rate of insulin from the blood into the pancreas.
• k_{31} = 1.75 U/hr be the flow rate of insulin from the pancreas into the blood.

Objectives
• Using the model, determine the flux equations for this system.
• Solve these equations for the concentration of insulin in each compartment.
• Plot the concentrations of insulin in each compartment for up to 10 hours.
• Given these results, estimate the steady-state level of insulin in the blood and the time at which this level is reached

In the previous hypotheses we have the following mathematical model:

blood	$x'_1=kin* x_4+k_{31}* x_3+k_{21}* x_2-k_{13}* x_1-k_{12}* x_1$
kidneys	$x'_2=k_{12}*x_1-k_{21} *x_2$
pancreas	$x'_3=k_{13}*x_1-k_{31}*x_3$
abdomen	$x'_4=-kin*x_4$