Project 29: PDE

Non linear Perona Malik diffusion for image denoising


1. Description of the model

Let's consider the anisotropic diffusion equation in 2-dimension:

$$u_t = d\Delta u + \nabla d \cdot \nabla u$$

Implement the denoising process on a grayscale image, in which the different gray levels are characterized by the function u = u (x, y, t). Perona Malik's model considers the nonlinear diffusion function:

$d(x,y,t) = \frac{1}{1+\frac{\|\nabla u\|^2}{K}}$, where K>0 is a suitable constant.

If we do not consider the term ∇d · ∇u in the second member of the equation, the problem is reduced to the two-dimensional heat equation. In this case the noise is removed from the image indiscriminately: there is a diffusion over the whole image. On the contrary, adding this term to the model allows you to preserve the contours, thus taking into account the structures of the image.

Determine the relationship between the proposed model and the PM model analyzed in the course.

Structure your code as follows:

- First you choose the image on which to apply the denoising process.
- Gaussian noise will then be applied to the original image (addnoise())
- You select the value for the constant K that appears within the diffusion function
- Then we move on to the choice of the discretization steps: the spatial step h is set equal to 1, and the time step tau is set at a sufficiently small value to guarantee the stability of the method (max tau = 0.25).
- You set the final time, that is the number of steps for integration.

For the resolution, implement an explicit upwind scheme for the advection term which could be added to the linear diffusion 2D code.
OPTIONAL implement the splitting in the two directions, solving two one-dimensional problems separately.

All your code should be integrated into the software provided during the exercises and compare with the other routines introduced that concern image manipulation according to different types of filters: 2D heat equation, Perona Malik's original algorithm, Total Variation filter. Evaluate the effectiveness in terms of error compared to the original image without noise.

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