

Project 59) BVP

Linear Boundary Value Problem

Consider the ordinary differential equation:

$$\begin{aligned} -u''(x) - 2u'(x) + e^x u(x) &= \sin(x)(1 + e^x) + \cos(x)(2e^{-x} - 1) \\ &\text{per } x \in [-\pi, \pi] \\ u(-\pi) = -e^\pi, \quad u(\pi) = -e^{-\pi}. \end{aligned}$$

with exact solution

$$u(x) = \sin(x) + e^{-x} \cos(x).$$

Solve the BVP problem using the shooting method and the finite difference (FD) method (function `bvpFD()`)

For the FD solution, approximate the first derivative by the three schemes: central diff. , backward difference. Forward difference. Using the `bvpFD ()` function, calculate the solution for each choice of the first derivative approximation method. Plot the discrete solutions together with the exact one and calculate the error.

OPTIONAL: Check the empirical order of convergence of the three methods.