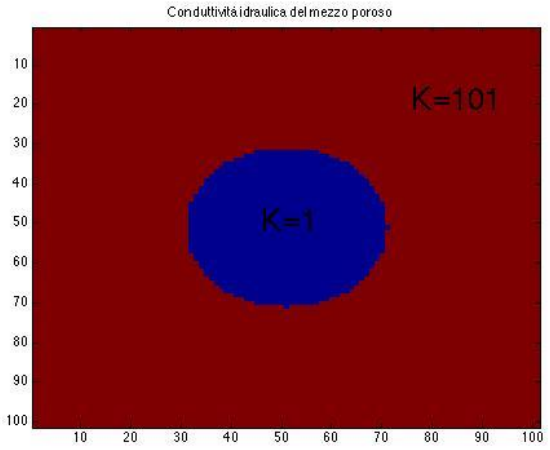
Project PDE 10

ANALYSIS OF THE PHENOMENON OF WATER FILTRATION
IN A POROUS MEDIUM



1. INTRODUCTION TO THE PROBLEM

The study of the filtration of a fluid in a porous medium is known as the Darcy problem. The real applications of this problem range from irrigation to the use and control of water resources, to the study of phenomena relating to cerebral circulation.

HALF POROUS is defined as a system consisting of solid granules between which there are spaces, called PORES, communicating with each other and with the outside. These pores form a series of interconnected ducts, inside which fluids, liquids and/or gases are contained.
The motion of a fluid in a porous medium is described with the term FILTRATION; the fluid moves in the network of irregular channels of the medium, meeting a high resistance so that the motion is generally very slow (it can last for days, months and sometimes even years).

The permeability of the porous medium is described through a constant, known as HYDRAULIC CONDUCTIVITY, which takes into account some characteristic properties of the medium such as porosity and size distribution of the granules.

2. DESCRIPTION OF THE MATHEMATICAL MODEL

Under appropriate hypotheses, the study of water filtration in a porous medium that occupies a two-dimensional region Ω can be investigated by the resolution of the following elliptic PDE problem

$$-\frac{\partial}{\partial x}\left(\mathrm{K}(x,y)\frac{\partial u}{\partial x}\right)-\frac{\partial}{\partial y}\left(\mathrm{K}(x,y)\frac{\partial u}{\partial y}\right)=0$$

where the function K is the hydraulic conductivity of the porous medium and u represents the variation of the water level in the medium.

In that case, the problem was solved by considering:

$$\mathrm{K}(x,y)=\begin{cases}101 & \text{if } (x-0.5)^2+(y-0.5)^2-0.04>0\\ 1 & \text{otherwise}\end{cases}$$

$$\Omega=(0,1)\times(0,1)$$

and imposing the following boundary conditions:

$$\begin{cases}\dfrac{\partial u}{\partial x}=0 & \text{on } (\{0\}\cup\{1\})\times(0,1)\\ u=10 & \text{on } (0,1)\times\{0\}\\ u=0 & \text{on } (0,1)\times\{1\}\end{cases}$$

Solve the elliptic PDE using finite difference schemes.