Project 9)
Model for the growth of tumors

Pennsyltucky Pharmaceutical is committed to trying to model the growth of certain types of tumors. On the basis of some simplifying hypotheses, it was able to determine the differential equation:

$$\frac{dR}{dt} = -\frac{1}{3} S_i R + \frac{2\lambda\sigma}{\mu R + \sqrt{\mu^2 R^2 + 4\sigma}}, \quad R(0) = a,$$

where R (t) indicates the radius of the tumor (assumed spherical); λ and μ are scale parameters, The rate at which cells in the center of the tumor die is measured and σ represents a nutritional level. To study this model, scientists at Pennsyltucky Pharmaceutical must solve this differential equation accurately for different sets of parameters.

(a) Solve the problem for λ = μ = 1, a = 0.25, Si = 0.8 and σ = 0.25. Use explicit and implicit single and multi-step methods to solve the problem and show that the tumor radius approaches a limit value for t that tends to infinity.

(b) Repeat with the parameters: λ = μ = 1, a = 0.50, Si = 0.9 and σ = 0.05, what happens in this case?

(c) We model some treatment for the tumor described by the given differential equation. In the previous exercises a constant nutritional level was assumed. Suppose we are able to decrease the nutritional level according to the model

$$\sigma(t) = \sigma_\infty + (\sigma_0 - \sigma_\infty)e^{-qt}$$

where σ_0 is the starting nutrient level, σ_{inf} the asymptotic nutrient level and q measures the rate at which the nutrient level decreases. Study the effect of various choices of these parameters on tumor growth, based on the observations in the previous points.