MODELING AND SIMULATION OF BUNGEE JUMPING

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AUTHOR'S CONTRIBUTION
This work was carried out by the sole author. Author SK gathered the initial data and interpret the results. Author SK designed the study, wrote the algorithm, methodology programming and interpreted the results. Author read and approved the final manuscript

Original Research. Article

ABSTRACT

In 1980, the bungee jump begun and started in UK. Where four athletes from "dangerous sport club" in Oxford University jumped from bridge in Bristol, UK toward the 250 foot Avon Gorge. Each attached one end of a nylon-braided, rubber shock cord to themselves and the other to the bridge. Nowadays bungee jumping is one of the most excited and extreme sports and the most thrilling one as well. Many tough guidelines and safety factors have set up to protect people practice this sport from any risk as much as possible. Unfortunately, even with all these precautions, bungee jumping accidents still occur. A recent accident happened in 2012 at Victoria falls, Zimbabwe. In this paper, we are going to present this problem, modeling then simulate it using Matlab software. We will use topics from ordinary differential equations theory and Physics science. A novel sensitivity analysis will be proposed at the end of this study. This paper can be used as reference to all engineering students as educational case study where the theory is applied to real life application, lead to a better understanding of the phenomena and maximize the human safety.

Keywords: Dynamic system; Newton law; Hooke law; differential equation; modeling; simulation; Matlab.

1 Introduction

Bungee jumping is a famous extreme sport that involves jumping from a high and fixed structure like bridge or building while attached to a large elastic cord. The bungee jumping is a typical example of dynamical system where physicists are interested to study and analyze its phenomenon. In brief, this experience can be described as: the bungee jumper jumps off a bridge and then falls down until the elastic bungee cord slows his descent to a stop, before pulling him back up. The jumper then oscillates up and down until all the energy is dissipated. The study of this sport in terms of safety and risk require using physics laws and theory of differential equations. In this paper, we will study completely this activity with different scenarios. Some simulations using Matlab is also given.

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2 History of Bungee Jumping

Bungee jumping activity came from a small Island in Australia [1]. A tribe called “Bunlap” practice this sport where the jump from towers. They used vine instead of cord. The Bunlap men practice this sport according to legend in this tribe to discover the trickery of their wife! In 1970, a reporter from National Geographic Magazine named Kal Muller went to this island and asked this tribe to teach him this activity. Later in 1979, four members of oxford danger sports read Muller’s stories. After that, they went to Clifton Bridge in Bristol, UK and jump using bungee cords instead of vines. Today bungee jumping is practicing worldwide [2-3].

3 Types of Bungee Jumping

There is several ways to jump, different types of bungee and many objects to jump from. Mainly, there are six types of bungee jumps fig. 1-6:

1- Forward jump

Fig. 1. Forward jump

2-Forward lumber jack

Fig. 2. Forward lumber jack

3-Backward jump

Fig. 3. Backward jump

4-Backward lumber jack

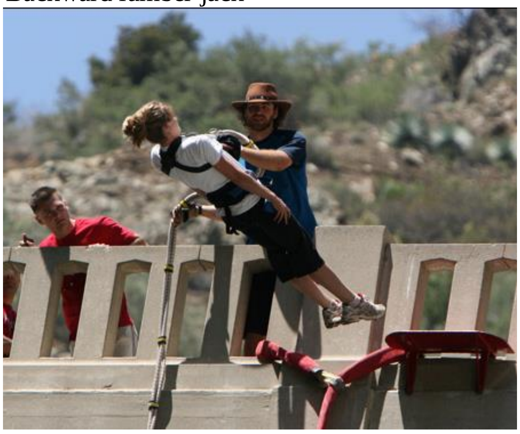

Fig. 4. Backward lumber jack

5-Elevator drop

Fig. 5. Elevator drop

6- Ankle dive

Fig. 6. Ankle dive

4 Case Study

In this case study we will study in depth and analyze a normal forward jump. The scenario is: Ahmed is standing on a bridge above a river. Ahmed, an industrial engineer, decides to jump off that bridge. His plan is to attach to his feet bungee cord, jump and pulled back gently before hit the river that is 350 meters (as shown in Fig. 7). He has to decide between several types of bungee cords, climbing rope and steel cable where they differ in their stiffness and length. The choice must avoid the water landing. In order to understand the problem, Ahmed decides to use two physics laws: Hooke and second law of Newton.

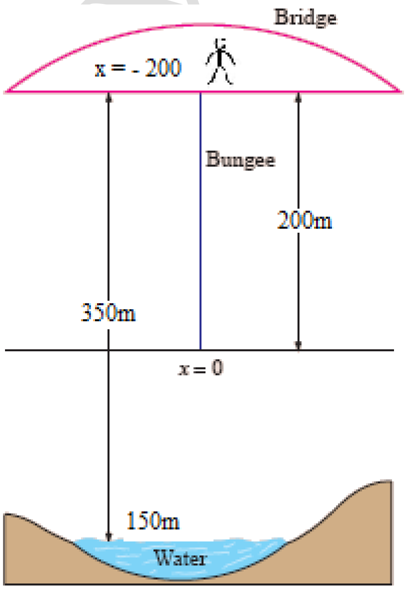

Fig. 7. Bungee jumping design

Before applying Newton second law [4] and Hooke law [5], we obtain:

$$\begin{aligned}
&F_H = -kx(t) \Rightarrow Hooke \\
&\sum F = mx''(t) \Rightarrow Newton \\
&\Rightarrow mg - cx'(t) - kx(t) = mx''(t)
\end{aligned} \tag{1}$$

Where (see table 1):

Table 1. numerical values of parameters

Symbol	Value	Description
m	80kg	Mass of Ahmed
x(t)	Varies with time (t)	Ahmed displacement
G	$9.8m/s^2$	Constant acceleration due to gravity
K	To be determined	Stiffness or rigidity of the cord
x'(t)	Varies with time (t)	Speed of Ahmed
x''(t)	Varies with time (t)	Acceleration of Ahmed
C	To be determined	Damping coefficient
x(0)	-200m	Height of Ahmed at time 0
x(tf)	350-200-2=148m	When head of Ahmed hits the water (Ahmed's tall 2m)

A. How to find the cord's stiffness?

In order to find the stiffness (k) of the cord which is constant, we attach an object of known mass (m) to the same cord and measure the length (h) between the original length (L) of the cord and total static stretch then we apply the law of energy conservation [6-8]:

$$k = \frac{2mgL}{h^2} \tag{2}$$

We found that k=12 N/m.

B. How to find the damping coefficient?

To find the damping coefficient (c), we use the free fall form of the equation (1):

$$mg - cx'(t) = mx''(t) \tag{3}$$

The terminal velocity [9], i.e. where $x''(t) = 0$, of a human body is $\cong$ 54 m/s.

Therefore $c = \frac{mg}{x'} = \frac{80x9.8}{54} = 14.5kg/s$

Solution of equation (1): part I (free fall)

C. Solution of equation (1): part I (free fall)

The purpose is to solve the free fall part of the second order non-homogenous differential equation in (1), in order to analyze the displacement x(t) (Fig. 8).

The equation (1) without damping, i.e. free fall part becomes:

$$mx''(t) + cx' = mg \Rightarrow 80x'' + 14.5x' = 784 \tag{4}$$

by using homogenous with constant coefficient technique and variation of parameters, we obtain:

$$x(t) = -5.52Ae^{-0.18t} + B + 54.07t \tag{5}$$

to find the constants A and B, we use the initial values

x(0)=-200 and x'(0)=0 ➔

$$x(t) = 298.32e^{-0.18t} + 54.07t - 498.31 \tag{6}$$

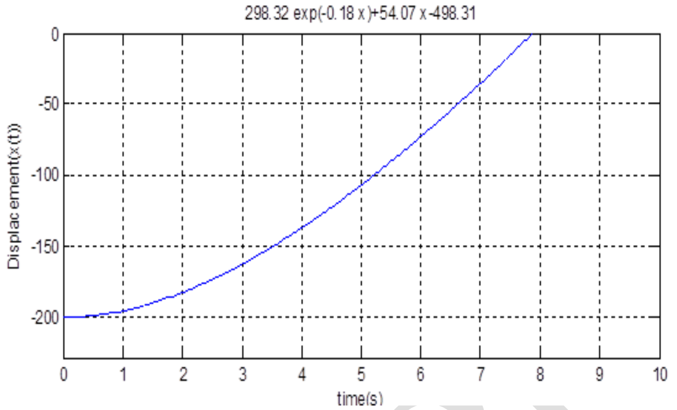

Fig. 8. Free fall displacement

We can find the time taken during the free fall. From the previous graph the time is between 7 and 8 seconds (when x'(t)=0). In order to find exactly the value, we have to solve numerically, by using Newton method for example, equation 6 for x(t) = 0 :

$$298.32e^{-0.18t} + 54.07t - 498.31 = 0 \tag{7}$$

Using Newton method [10] in Matlab the exact answer is 7.88 seconds, i.e. x(7.88)=0.

To find the velocity when the cord starts pull back, i.e. at the end of the free fall distance (Fig. 9), we can derive equation (3) at t=7.88s, $-53.69e^{-0.18x7.88}+54.07$ = 41.07m/s.

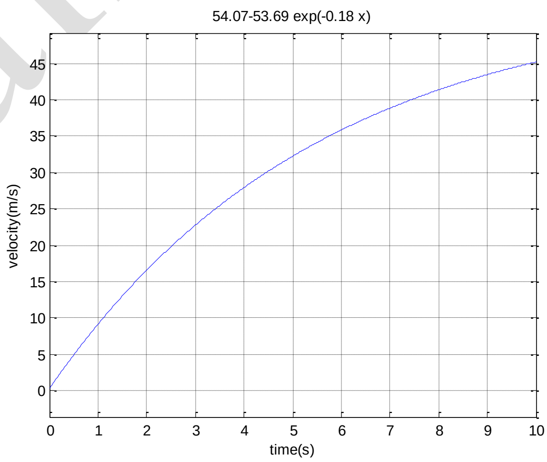

Fig. 9. Velocity

D. Solution of equation (1): part II (vibration)

We will solve the second part of the bungee jumping which is the vibration part (Fig. 10). In this part will study and analyze the behavior of Ahmed after traveled the natural length of the cord (200m). We use same technique to solve $80x''(t)+14.5x'(t)+12x(t)=784$:

$$x(t)=Ae^{-0.09t}\sin(0.37t)+Be^{-0.09t}\cos(0.37t)+65.33$$

By using the initial values x(7.88)=0, x'(7.88)=41.07:

$$x(t)=-210.91e^{-0.09t}\sin(0.37t)+98.33e^{-0.09t}\cos(0.37t)+65.33 \quad (8)$$

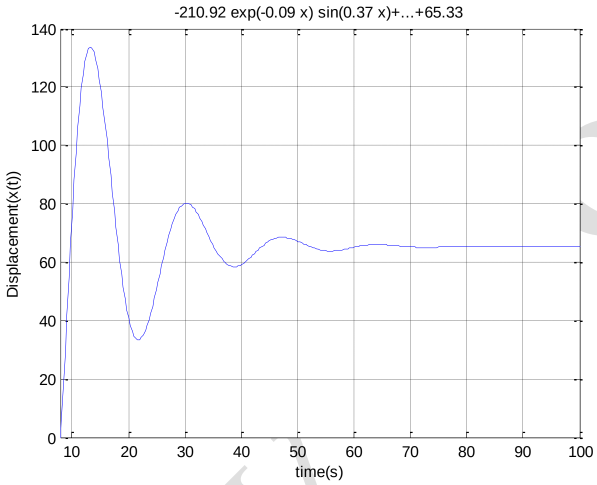

Fig. 10. Vibration analysis

We must find the "time" Ahmed spent to stops descending, in other word "the lowest position below the bridge". This value will help us to check if Ahmed reaches the water. To find this value, we can find the value of t where the derivative of equation (3), the displacement, is zero. Using Matlab and global maxima for t>7.88, we obtain x(t)=134m for t=13.3s, so the lowest point can Ahmed rich it is 134+200+2=336m<350m. By using the selected cord, Ahmed will be safe and do not hit the water.

5 Sensitivity Analysis of Stiffness

Cords generally do not behave like linear springs over their entire range of use [11-14], therefore incorporate variable stiffness in the bungee cord give more realistic situation. The stiffness varies with the elongation of the cord (Fig. 11). For illustration purpose, we will evaluate the stiffness for three different weights.

The elongation is not linear therefore different value of the stiffness (Fig. 12) must be taken into consideration instead of constant value. In this simulation, we will use three different values for the stiffness k as follows:

$$\begin{cases} k=13.72, \text{ if } \ x\le 25.5 \\ k=12, \text{ if } \ 25.5<x\le 35.33 \\ k=16.66, \text{ if } \ x>35.33 \end{cases} \quad (9)$$

The solution of the differential equation will be:

$$x(t)=\begin{cases} -176.27e^{-0.09t}\sin(0.4t)+124.33\,e^{-0.09t}\cos(0.4t)+ \\ 57.14,\ if\ \ t\le 25.5 \\ -210.91e^{-0.09t}\sin(0.37t)+98.32e^{-0.09t}\cos(0.37t)+ \\ 65.33,\ if\ \ 25.5<t\le 35.33 \\ -120.01e^{-0.09t}\sin(0.44t)+151.93e^{-0.09t}\cos(0.44t)+ \\ 47.05,\ if\ \ t>35.33 \end{cases} \tag{10}$$

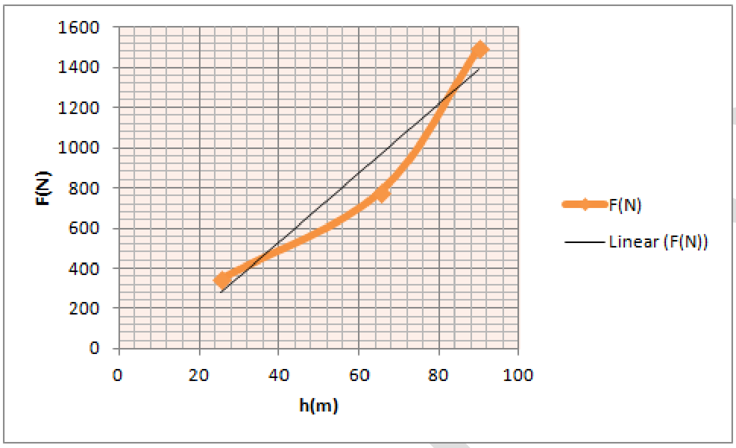

Fig. 11. Non linear stiffness

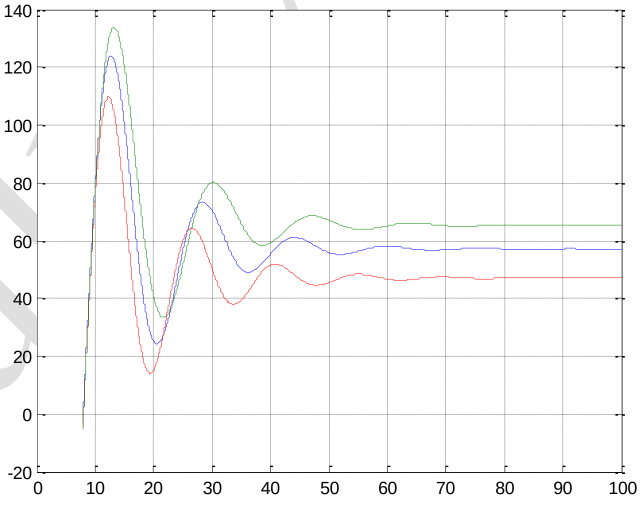

Fig. 12. Displacement for non linear stiffness

6 Conclusion

In this educational study, we use and apply different courses. Basically, we used physics to understand the forces laws, differential equation to solve the dynamic behavior of the jumper, numerical methods to solve numerically some complex equations. This study can be further extending it by developing software to simulate different types of stiffness and damping coefficients against the jumper weight. A sensitivity analysis of the damping factor might be also studied.

Competing Interests

Authors have declared that no competing interests exist.

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