Project 12:

EXCHANGE OF 0_2 IN THE LUNG CAPILLARIES

1. DESCRIPTION OF THE PROBLEM

At the basis of the phenomenon of respiratory gas exchange at the level of the pulmonary capillaries, there is the ability of the gases to move from one point to another by diffusion, due to a difference in partial pressure. Thus, oxygen diffuses from the alveoli into the pulmonary capillaries due to a difference in pressure, since the partial pressure of oxygen (P_{O_2}) in the alveoli is higher than that of the pulmonary blood. The difference between P_{O_2} pulmonary capillary and alveolus varies along the capillary, going from a value of about 64 mmHg at the proximal end of the capillary to about 0 mmHg at the distal end. During quiet breathing the mean difference (in time and space) is about 11 mmHg. Starting from one P_{O_2} in the venous blood that reaches the lungs, equal to about 40 mmHg, it is thus possible to obtain a P_{O_2}, in the blood that leaves the capillaries, equal to about 104 mmHg. The P_{O_2} in the blood of the capillaries becomes approximately equal to that of the alveolar (104 mmHg) within the first third of the length of the capillary. The main obstacle in determining the variation P_{O_2} along the pulmonary capillary is represented by the non-linear dependence of the gas concentration on the partial pressure; it is precisely this non-linearity that prevents the analytical resolution of the equation underlying the problem.

2. MATHEMATICAL MODEL

The trend of P_{O_2} along the pulmonary capillary is described by the following ordinary differential equation, obtained through a mass balance applied to the infinitesimal segment dx:

$$\frac{dC_{O_2}}{dP_{O_2}}\frac{dP_{O_2}(x)}{dx} = -\frac{2\pi R D_{O_2}}{Q}\left(P_{O_2}(x) - P_{A_{O_2}}\right)$$

where

C_{O_2} = gas concentration in the capillary

P_{O_2} = partial pressure of the gas in the capillary

R = radius of the capillary

$$D_{O_2} = \frac{diffusion\ coefficient}{thickness\ of\ \ the\ capillary\ wall}$$

Q = blood flow

P_{AO_2} = partial pressure of the gas in the alveolar space

The relation that binds C_{O_2} to P_{O_2} can be well described by the so-called Hill equation:

$$\frac{C_{O_2}}{C_{O_2\,sat}} = \frac{\left(P_{O_2}/P_0\right)^n}{1+\left(P_{O_2}/P_0\right)^n},$$

where

$C_{O_2\,sat}$ = max value of C_{O_2}

P_0 = 27.2 mmHg (value such that $C_{O_2} = C_{O_2\,sat}/2$, 50% saturation)

n = 2.7

P_0 and n were determined empirically in order to approximate the curve measured experimentally as closely as possible.

By calculating the derivative of C_{O_2} respect to P_{O_2}, based on the relationship provided by the Hill equation, and substituting in the differential equation of the model, we obtain:

$$\frac{dP_{O_2}(x)}{dx} = -\frac{2\pi R D_{O_2} P_0^{\,n}}{QnC_{O_{2sat}}}\left(\frac{P_{O_2}(x)-P_{A_{O_2}}}{P_{O_2}(x)^{n-1}}\right)\left(1+\left(\frac{P_{O_2}(x)}{P_0}\right)^n\right)^2$$

Solve the problem numerically by considering the following values for the parameters:

R = 3.5 μm

D_{O_2} = 0.538 mL/m^2/(min mmHg)

$C_{O_2\,sat}$ =20 mL O_2/100 mL blood

$Q = CO/N$, CO = 6 L/min (total blood flow to the lungs)

N =2.6*10^9 (number of open pulmonary capillaries)

P_{AO_2} =104 mmHg

Length of the capillary = 650 μm

and imposing the following initial condition: $P_{O_2}(0)$=40 mmHg