Exercises
Università di Bologna, May 2012

Exercise 9:

In the real projective space P^2 four points E_1, E_2, E_3, E are given by the vectors

$$\vec{e}_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \ \vec{e}_2 = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}, \ \vec{e}_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \ \vec{e} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix},$$

a) Verify that the points E_1, E_2, E_3 and E are in general position.

b) Choose $\{E_1, E_2, E_3; E\}$ as projective coordinate system and determine the projective coordinates of the point X with respect to this coordinate system, where X is given by the vector $\vec{x} = (4, 2, 3)^T$.

Exercise 10:

Let $\{A_1, A_2, A_3; A\}$ be a projective coordinate system in the real projective plane P^2 with respect to which the four points E_1, E_2, E_3, E have the projective coordinates $E_1(9, 4, 1)$, $E_2(27, 8, 1)$, $E_3(3, 2, 1)$, $E(0, -6, -4)$.

a) Show that it is possible to introduce $\{E_1, E_2, E_3; E\}$ as new projective coordinate system in P^2.

b) In the projective coordinate system $\{A_1, A_2, A_3; A\}$ the point X has the coordinates (x_1, x_2, x_3). Determine the projective coordinates (y_1, y_2, y_3) of X in the projective coordinate system $\{E_1, E_2, E_3; E\}$.

Exercise 11:

In a coordinate system in the Euclidean plane the points P_i, Q_i have the coordinates:

$$P_1(0, -1)\,,\ P_2(-1, 0)\,,\ P_3(0, 1)\,,\ P_4(1, 0)\,,\ P_5(0, 0)$$

$$Q_1(1, 1)\,,\ Q_2(1, 3)\,,\ Q_3(3, 3)\,,\ Q_4(3, 1)\,,$$

a) Represent the situation graphically.

b) Write the points P_i and Q_i in homogeneous coordinates.

c) Show that there exists one and only one projective map $\pi : P^2 \longrightarrow P^2$ with $\pi(P_i) = Q_i$ for $i = 1, \ldots, 4$. Find its matrix. Tip: Make projective coordinate systems out of the points $P_1, \ldots, P_4$ respectively $Q_1, \ldots, Q_4$.

d) Determine the image point Q_5 of the point P_5 with respect to π.

Exercise 12:

Let $B_0(\overrightarrow{b_0}), B_1(\overrightarrow{b_1}), B_2(\overrightarrow{b_2})$ be the control points of a parabola, where $\overrightarrow{b_0} = (0,0)^T, \overrightarrow{b_1} = (-1,5)^T, \overrightarrow{b_2} = (6,2)^T$. Determine the parameter representation of the parabola $\overrightarrow{x}(t)$ defined by the points B_0, B_1, B_2, on the interval $[0,1]$. Compute the curve point corresponding to the parameter value $t = 1/4$ by using de Calsteljau's algorithm. Illustrate graphically.

Exercise 13:

Let

$$\overrightarrow{x}(t) = \frac{\sum_{i=0}^{2} w_i \overrightarrow{b}_i B_i^2(t)}{\sum_{i=0}^{2} w_i B_i^2(t)}$$

where $\overrightarrow{b}_0 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$, $\overrightarrow{b}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, $\overrightarrow{b}_2 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$, $w_0 = 1$, $w_1 = w_2 = 4$, be a conic section in Bézier form. Determine its type.

Exercise 14:

Let c be a circle with center C(1,1) and radius 1, and let $B_0(1,2), B_2(2,1) \in c$ be two points of this circle (where $B_0(\overrightarrow{b}_0), B_2(\overrightarrow{b}_2)$).

Determine the point $B_1(\overrightarrow{b}_1)$ and the weight w_1 such that the rational Bézier segment

$$\overrightarrow{x}(t) = \frac{\overrightarrow{b}_0 B_0^2(t) + w_1 \overrightarrow{b}_1 B_1^2(t) + \overrightarrow{b}_2 B_2^2(t)}{B_0^2(t) + w_1 B_1^2(t) + B_2^2(t)}$$

describes a circular arc c.