Mechanics of growth: from physics to geometry

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The study of bodies that can grow (gain or lose mass) and that can remodel internally is a rapidly evolving and interdisciplinary field, posing fundamental challenges in both modeling and applications. Growth and remodeling span a wide range of scales and materials, from gravitational accretion to the formation of sandpile, from the manufacturing of masonry structures to biological growth, from technological processes like coil winding to additive manufacturing. A central aspect of all of these natural or technological processes is the accumulation of residual stresses, which can either lead to failure, or be harnessed to design materials with novel functionalities.

This mini-course will introduce the fundamental principles governing the mechanics of growing bodies, focusing on the emergence of non-Euclidean features in their stress-free configuration and on the differential geometric framework underlying strain incompatibility, the main character behind residual stress accumulation. We will analyze how incompatibility, which is measured by the Ricci tensor of the growth metric, arises and is maintained during growth and remodelling, distinguishing between two primary mechanisms of "surface growth", where new material is deposited onto the boundary of an existing body, and of "volumetric growth", where mass accretion and reorganization occur internally.

To accurately describe growth-induced residual stresses and the inelastic phenomena responsible for incompatibility, a richer kinematics beyond classical Euclidean continua is required. This advanced mathematical description is essential for understanding and designing new materials that encode additional "information", unlocking unprecedented mechanical properties beyond those of conventional materials.