EACWE 5
Florence, Italy
19th – 23rd July 2009

Flying Sphere image © Museo Ideale L. Da Vinci

A numerical procedure for the dynamic response of tall buildings subject to turbulent wind excitation

G. Barbella[*], F. Perotti[*], V. Simoncini[**]
[*]*Department of Structural Engineering, Politecnico di Milano, Italy,*
e-mail: barbella@stru.polimi.it, perotti@stru.polimi.it
[**]*Department of Mathematics, University of Bologna, Italy,*
e-mail: valeria@dm.unibo.it

Keywords: tall buildings, gust buffeting, iterative methods.

ABSTRACT

One of the most challenging issues in the modern design of tall buildings is related to the evaluation of wind actions, which can be, in some cases, much more demanding in terms of ultimate resistance and serviceability conditions behaviour when compared to seismic actions. The current normative framework (Euro Code 1-4), based on simplified computing methodologies mostly consisting of equivalent static analyses, seems to be inadequate – and thus not applicable – for complex and/or sensitive structures, or when basic regularity requirements are not fulfilled. In these cases, more refined methods are recommended, even though without precise references or guidelines.

On the other hand, the increasing availability and accuracy of wind tunnel tests data, allows to obtain a sophisticated description of the aerodynamic behaviour of the structure, which can be exploited, together with a finite element mechanical model, in order to evaluate the dynamic structural response to gust buffeting phenomena. This can be done by considering two different procedures: the first makes use of the whole set of time history recordings, coming from wind tunnel, as forcing terms to be applied to the FE model in a deterministic context; the second consists of a stochastic approach, based on a probabilistic model of the wind turbulence, as well as on a suitable model describing the fluid-structure interaction, characterized by the aerodynamic coefficients computed from the wind tunnel tests.

The present work mostly addresses the latter approach, by proposing a numerical procedure which is intended to somehow plug the gap in terms of design prescriptions. Nonetheless, the deterministic problem can be properly handled within the same mathematical framework, giving the opportunity to carry out a complete buffeting analysis of the structure. In both cases an important role is assumed for the evaluation of higher vibration modes contribution and for the verification of serviceability conditions related to comfort.

Fluid-structure interaction forces are computed according to the Morison approach, which implies some base hypotheses: body motion does not influence upstream fluid velocity field; interaction forces depend only on fluid-structure relative motion; aerodynamic coefficients are determined

Contact person: G. Barbella, Department of Structural Engineering, Politecnico di Milano
E-mail barbella@stru.polimi.it

through measurements in stationary regime on a fixed body; friction effects are neglected. Then let v_1, v_2 and v_3 be the wind reference axes, with v_1 parallel to the mean velocity, v_2 orthogonal to the first and lying on the horizontal plane and v_3 in the vertical direction; let W be the wind mean velocity and w_i the turbulence components relative to the v_i axes. The local force per unit length acting on an element with the axis perpendicular to v_1, written in the wind reference v_1v_2 and neglecting inertia contribution, assumes the form

$$\mathbf{p} = \begin{Bmatrix} p_{v_1} \\ p_{v_2} \\ p_{\theta_3} \end{Bmatrix} = \frac{1}{2}\rho D V_{\mathrm{r}}^2 \begin{Bmatrix} C_{\mathrm{D}}\cos\gamma_{\mathrm{r}} - C_{\mathrm{L}}\sin\gamma_{\mathrm{r}} \\ C_{\mathrm{D}}\,\mathrm{sen}\,\gamma_{\mathrm{r}} + C_{\mathrm{L}}\cos\gamma_{\mathrm{r}} \\ D C_{\mathrm{M}} \end{Bmatrix}, \tag{1}$$

where ρ, D, V_{r} and γ_{r} are respectively the air density, the characteristic transversal dimension of the element, the modulus of the fluid-structure relative velocity and the angle of attack fluctuation; while C_{D}, C_{L} and C_{M} are the drag, lift and moment aerodynamic coefficients. Linearization of expression (1) is carried out considering that structure displacements are assumed to be small and, in characteristic design conditions, mean value of wind velocity is much larger than both its fluctuations and structure velocities. As a result, interaction forces are subdivided into the static, dynamic, damping and stiffness contribution, respectively depending on mean wind velocity, turbulence components, structure velocities and structure displacements. Finally, distributed wind loads are applied to the structure by considering two-node "aerodynamic" linear elements, ideally representing the axis of the slender body immersed in the fluid flow. Such an approach, in the case of tall buildings, can be adopted under the hypothesis of in-plane rigid floor diaphragms, so that aerodynamic elements are connected to the centres of mass of adjacent floors. It has to be noticed that the aerodynamic contributions to viscous damping and stiffness matrices, lead to the loss of symmetry properties.

The three-dimensional description of wind turbulence is formulated according to the current code regulation parameters and to the stochastic model proposed by Piccardo and Solari (1998), based on the hypothesis of stationarity for the turbulence process. The cross-spectral power density of the wind fluctuations components w_j and w_k', considered in the points $P \equiv \{x, y, z\}$ and $P' \equiv \{x', y', z'\}$, is computed as

$$S_{jk}(P,P',f) = \sqrt{S_j(z,f)\,S_k(z',f)}\;\mathrm{Coh}_{jk}(P,P',f), \tag{2}$$

where S_j and S_k are the auto-spectra of the single components; while the coherence function accounts for both the correlation Γ between different velocity fluctuations, along directions j and k, in the same point, and the spatial correlation Λ between fluctuation components relative to the same direction but considered in different points:

$$\mathrm{Coh}_{jk}(P,P',f) = \mathrm{Sgn}\left(\Gamma_{jk}\right)\sqrt{\Gamma_{jk}(z,f)\,\Gamma_{jk}(z',f)}\sqrt{\Lambda_j(P,P',f)\,\Lambda_k(P,P',f)}\,. \tag{3}$$

Structural response is computed starting from the dynamic equilibrium equation in the frequency domain, which has the form

$$\left[-(2\pi f)^2\mathbf{M} + i2\pi f(\mathbf{C}_{\mathrm{v}} + \mathbf{C}_{\mathrm{a}}) + \mathbf{K} + \mathbf{K}_{\mathrm{a}} + i\,\mathbf{C}_{\mathrm{h}}\right]\tilde{\mathbf{q}}(f) = \mathbf{E}(f)\,\tilde{\mathbf{q}}(f) = \tilde{\mathbf{p}}(f) = \mathbf{F}\tilde{\mathbf{w}}(f), \tag{4}$$

where $\mathbf{E}(f)$ is the impedance matrix of the system, while $\tilde{\mathbf{q}}(f)$ and $\tilde{\mathbf{p}}(f)$ are the Fourier Transforms (FT) of structural displacements and of the generalized components of external actions. Here damping, stiffness and dynamic contributions of aerodynamic forces are accounted for in the assembling procedure of system matrices and right hand side vectors. Displacements spectral power densities matrix is defined as

$$\mathbf{S_Q}(f) = \lim_{T\to\infty}\frac{1}{T}E\left[\tilde{\mathbf{q}}(f,T)\,\tilde{\mathbf{q}}^{\mathrm{H}}(f,T)\right] = \mathbf{E}^{-1}(f)\,\mathbf{S_P}(f)\,\mathbf{E}^{-\mathrm{H}}(f) = \mathbf{H}(f)\,\mathbf{S_P}(f)\,\mathbf{H}^{\mathrm{H}}(f), \tag{5}$$

in which $E[\bullet]$ is the expected value operator, $\mathbf{S_P}(f) = \mathbf{F}\,\mathbf{S_W}(f)\,\mathbf{F}^{\mathrm{T}}$ and $\mathbf{S_W}(f)$ is the SPD matrix of wind turbulence components, whose single entries take the form of expression (2). In the deterministic case, as already mentioned, global external wind actions are assumed to be directly computed from wind tunnel pressure measurements, hence, impedance matrix keeps being symmetric. In fact, in such a case damping contributions due to the fluid-structure relative motion, which cannot be measured in wind tunnel aerodynamic tests, may be accounted for by adding suitable viscous dampers to the mechanical model.

Expression (5) is computed by considering the non-symmetric $N \times N$ complex linear system with multiple right hand sides

$$\mathbf{E}(f)\,\mathbf{X_q}(f) = \mathbf{F}\,, \tag{6}$$

which has to be solved for each value of the frequency parameter. Thus, the use of a direct solver would imply significant costs in terms of computing resources, since impedance matrix is a function of frequency and a new factorization must be performed at each step.

The numerical procedure here adopted is based on iterative schemes which allow for the simultaneous solution of the system for several frequency steps, see (Feriani & al., 2000) and (Simoncini and Perotti, 2002). The standard algebraic formulation of the problem is derived rewriting (4) in terms of acceleration FT, hence, equation (6) becomes:

$$\left[\mathbf{M} + \frac{\mathbf{C}}{i2\pi f} - \frac{\mathbf{K}}{(2\pi f)^2}\right] \mathbf{X_a}(f) = \mathbf{F}\,, \tag{7}$$

which can be linearized with respect to the frequency parameter $\lambda = (2\pi f)^{-1}$, providing the shifted system of dimension $2N$

$$\left\{\mathcal{A}\mathcal{B}^{-1} + \lambda \mathbf{I}_{2N}\right\}\mathbf{z} = \mathbf{d}\,,\ \text{with}\ \mathcal{A} = \begin{bmatrix} i\mathbf{C} & -\mathbf{M} \\ -\mathbf{M} & \mathbf{0} \end{bmatrix},\ \mathcal{B} = \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix},\ \mathbf{z} = \mathcal{B}\begin{bmatrix} -\lambda\tilde{\mathbf{a}} \\ -\tilde{\mathbf{a}} \end{bmatrix},\ \mathbf{d} = \begin{bmatrix} \mathbf{F} \\ \mathbf{0} \end{bmatrix}. \tag{8}$$

Note that system (8) is not symmetric, even though both $\mathcal{A}$ and $\mathcal{B}$ come out to be symmetric, when for example the deterministic case is considered.

The iterative solvers here adopted (Saad, 1996) rely on projection methods onto Krylov subspace, meaning that, given the generic non-Hermitian complex linear system $\mathbf{Ax} = \mathbf{b}$, an approximate solution $\mathbf{x}_m \approx \mathbf{A}^{-1}\mathbf{b}$ is obtained by searching $\mathbf{x}$ in the Krylov subspace generated by the coefficients matrix and a starting vector $\mathbf{v}$, defined as

$$\mathcal{K}_m(\mathbf{A},\mathbf{v}) = \operatorname{span}\left\{\mathbf{v}, \mathbf{Av}, \mathbf{A}^2\mathbf{v}, \ldots, \mathbf{A}^{m-1}\mathbf{v}\right\}. \tag{9}$$

In the implementation, $\mathbf{v}$ is usually set equal to the starting residual vector $\mathbf{r}_0 = \mathbf{Ax}_0 - \mathbf{b}$, where $\mathbf{x}_0$ is an initial guess. The key feature of this strategy is that the generated subspace is invariant under shift, that is $\mathcal{K}_m(\mathbf{A},\mathbf{v}) = \mathcal{K}_m(\mathbf{A}+\lambda\mathbf{I},\mathbf{v})$, so that the same search subspace is used for all the considered shifts. Different methods are characterized by the procedure adopted to build a basis for $\mathcal{K}_m$ and to minimize the difference between the approximate and the exact solution. The first algorithm implemented in this work is the GMRES method, based on the Arnoldi process, which provides an orthonormal basis $\mathbf{V}$ and an upper Hessenberg matrix, containing the orthogonalization coefficients. Note that orthogonalization of the basis vectors has to be performed against all the previous ones, so that the entire matrix $\mathbf{V}$ need to be stored. Then, the algorithm updates the approximate solution by minimizing the 2-norm of the associated residual, which requires the solution of a least square problem for all the shifts at each iteration, and implies a monotonic convergence as m increases. Moreover, in order to conveniently handle problem (8), with multiple r.h.s., suitable block versions of the method have to be considered, rather than solving the different systems separately. Thus, the Krylov subspace assumes the form:

$$\mathcal{K}_{m\times s}(\mathbf{G},\mathbf{r}_0) = \operatorname{span}\left\{\mathbf{r}_0, \mathbf{Gr}_0, \mathbf{G}^2\mathbf{r}_0, \ldots, \mathbf{G}^{m-1}\mathbf{r}_0\right\}\,,\quad \mathbf{r}_0 = \mathbf{F}\,, \tag{10}$$

with $\mathbf{G} = \mathcal{A}\mathcal{B}^{-1}$, s being the number of r.h.s. and having set the initial guess to zero. The main drawback of the block variant of Krylov subspace projection methods lies in the memory storage requirements, since solution vectors, if the shifts and r.h.s. numbers are large, cannot be stored to be iteratively updated. Hence, it can be convenient just to save the minimization coefficients and the basis vectors; also, one needs to store Givens' rotations for the QR factorization of Hessenberg matrices. As for the need to orthogonalize the entire matrix **V**, short-term recurrence in the subspace building process can be considered. In particular, a block-QMR method has been implemented, based on the two-sided Lanczos algorithm, which requires the explicit creation of two subspaces: one associated with **G** and the other with its transpose. Moreover, when the matrix **G** comes out to be *J*-symmetric, the two-sided Lanczos algorithm reduces into the simplified Lanczos – transpose free – variant. However, the application of such simplification is possible only for the deterministic case.

A computing example of the presented methodologies is shown, with reference to a 70 meters high real building, for which a complete wind tunnel test campaign has been carried out. The mechanical finite element model is composed by 3D beam elements, with 2364 dofs. The computing times, in seconds, for the block-GMRES method are 6.675, 8.459 and 10.590, if the number of frequencies is 11, 101 or 201 respectively. Figure 1 depicts some of the results from the stochastic

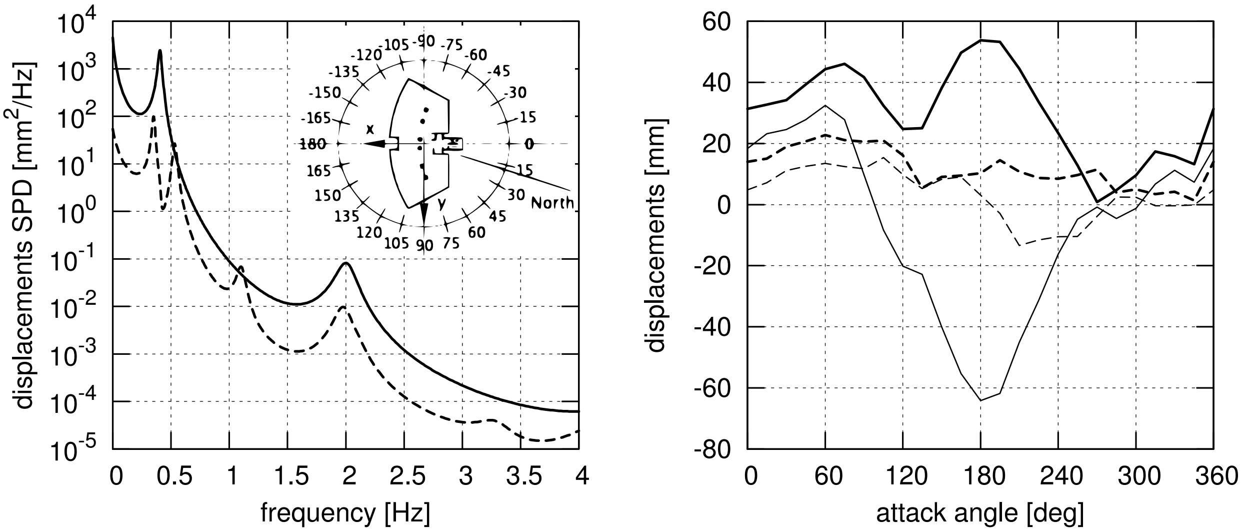

Figure 1: left – displacements spectral power densities for the central node of the highest floor in x (solid) and y (dashed) directions, angle of attack is 180° (with v_1 parallel to x); right – extreme (thick) values of displacements from the stochastic analysis, compared to static (thin) values in x (solid) and y (dashed) directions.

analysis: on the right, displacements of the last floor, due to the wind static component, are compared with their extreme values for each angle of attack; on the left, spectral power densities of displacements in the horizontal plane are reported, corresponding to the most demanding load combination case.

REFERENCES

Piccardo, G. and Solari, G. (1998), "Closed form prediction of 3-D wind-excited response of slender structures", Wind Engineering and Industrial Aerodynamics, vol. 74–76, pp. 697–708.

Feriani, A., Perotti, F. and Simoncini, V. (2000), "Iterative system solvers for the frequency analysis of linear mechanical systems", Computer Methods in Applied Mechanics and Engineering, vol. 190, pp. 1719–1739.

Simoncini, V. and Perotti, F. (2002), "On the numerical solution of $(\lambda^2 A + \lambda B + C)x = b$ and application to structural dynamics", SIAM Journal on Scientific Computing, vol. 23, no. 6, pp. 1875–1897.

Saad, Y. (1996), "Iterative methods for sparse linear systems", PWS Publishing Company.