CIME SUMMER SCHOOL

Exploiting Hidden Structure in Matrix Computations. Algorithms and Applications

June 21-26, 2015, Cetraro, Italy

Directors of the school: Michele Benzi and Valeria Simoncini

Motivation. Matrices with special structure arise frequently in scientific and engineering applications, and have long been object of study in numerical linear algebra, as well as in matrix and operator theory. Efficient structure-exploiting algorithms have been devised for solving a wide range of problems for these types of matrices, such as linear and nonlinear systems and (polynomial) eigenvalue problems. An enormous literature exists on Toeplitz matrices and their generalizations. Matrices with semi-separable or quasi-separable structure have also been intensely investigated. Another important example is that of Hamiltonian and symplectic matrices, for which stable and efficient structure-preserving algorithms have been developed. These matrices arise in optimal control problems for both classical and quantum mechanical systems. In all these examples, the matrix structure in question is more or less transparent; it may be determined by an inspection of the pattern of the matrix entries, or from a priori knowledge of the underlying physics.

More recently, researchers have begun to investigate classes of matrices for which structural properties, while present, may not be immediately obvious. An important example is that of matrices associated with complex networks, such as the adjacency matrix or the graph Laplacian associated with a social network. On the surface, these matrices may seem devoided of any useful structure; upon closer inspection, however, they often have a great deal of latent structure, for instance the nonzero and eigenvalue distribution tend to obey very precise laws, typically power laws. Knowledge of these properties can be of great importance in the design of efficient numerical methods for such problems.

In other cases the matrix is only “approximately” structured; for instance, a matrix could be close in some norm to a matrix that has a desirable structure, such as a banded matrix or a semiseparable matrix. In such cases it may be possible to develop solution algorithms that exploit the “near-structure” present in the matrix; for example, efficient preconditioners could be developed for the nearby structured problem and applied to the original problem. In other cases the solution of the nearby problem, for which efficient algorithms exist, may be a sufficiently good approximation to the solution of the original problem, and the main difficulty could be detecting the “nearest” structured problem.

Aim of the school. We propose a set of lectures focusing on the topic of “approximately structured” matrix problems and their efficient numerical solution.

1. Matrices with decay (“approximately banded or sparse matrices”)

2. Matrices which are approximately Kronecker products/sums;
3. Low-rank approximations of dense matrices (inverses, Schur complements, matrix functions, etc)
4. Matrices with special nonzero and eigenvalue distributions
5. Parameterized matrices
6. Applications (complex networks, quantum chemistry, matrix functions, matrix equations, stochastic PDEs, preconditioning)

Prerequisites: Matrix Analysis, Numerical linear algebra.

List of speakers:

(1) **Michele Benzi** (`http://www.mathcs.emory.edu/~benzi/`)
Samuel Candler Dobbs Professor, Department of Mathematics and Computer Science, Emory University (USA).

(2) **Dario A. Bini** (`http://www.dm.unipi.it/~bini/`)
Professor of Numerical Analysis, Dipartimento di Matematica, Università di Pisa, Italy.

(3) **Daniel Kressner** (`http://people.epfl.ch/daniel.kressner`)
Associate Professor, Numerical Algorithms and High-Performance Computing, EPFL Lausanne.

(4) **Hans Munthe-Kaas** (`http://www.ii.uib.no/~hans/work/Contact.html`)
Professor, Department of Mathematics, University of Bergen, Norway.

(5) **Charles Van Loan** (`http://www.cs.cornell.edu/cv/`)
Joseph C. Ford Professor of Engineering, Department of Computer Science, Cornell University, USA.

Michele Benzi, Department of Mathematics and Computer Science, Emory University, USA
E-mail address: `benzi@mathcs.emory.edu`
URL: `http://www.mathcs.emory.edu/~benzi/`

Valeria Simoncini, Dipartimento di Matematica, Università di Bologna, Italy
E-mail address: `valeria.simoncini@unibo.it`
URL: `http://www.dm.unibo.it/~simoncin/`